Validation of the Approximate Time-domain Method for the Lightning-horizontal Electric Field at the Surface of Two-layer Earth by Using FDTD

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Abstract—Based on the 2-D FDTD method, we have validated the time-domain method presented by Barbosa et al. (B-P formula) for calculating the lightning-radiated horizontal electric field at the earth surface within distances from 30 m to 1000 m from the lightning channel, considering the horizontally stratified earth. The results show that, except for distances in the vicinity of the lightning channel (e.g., 30 m), the B-P approximate formula can approximately predict the field peak value with a satisfactory accuracy within errors of 10% from 50 m to 1000 m from the lightning channel for the subsequent return strokes. The rise-time part of the field wave predicted by the B-P approximate formula is similar to that predicted by 2-D FDTD, however, the tail of the wave predicted by the B-P approximation may be shorter than that by the 2-D FDTD (e.g., 30 m and 50 m).

Keywords-2-D FDTD, lightning horizontal field, stratified earth, lightning return stroke.

I. INTRODUCTION

The lightning horizontal electric field is very important for the accurate estimation of the lightning-induced voltages on overhead lines. The exact solution for this problem of finitely conducting ground can be evaluated by numerical integration of Sommerfeld’s integrals [1], which are improper integrals with highly oscillating integrand functions. In order to overcome the difficulty, a number of numerical techniques have been proposed [2]-[8]. By using the surface impedance concept, Cooray [9] developed a frequency-domain expression. Rubinstein [10] extended the Cooray’s approach to evaluate the horizontal electric field at a given height above ground, which is known as the widely used C-R formula. He found that the C-R formula is suitable for evaluate lightning horizontal electric field at close, intermediate and far range [11]. In the last two years, the C-R approximate formula has been extended to the horizontally and vertically stratified ground [12], [13]. Recently, Zhang et al. [14] have further extended the C-R formula into a rough and ocean land mixed propagation surface and analyzed the propagation effect of the roughness of the ocean surface and land section on the lightning-radiated horizontal field. In order to validate the accuracy of the extended C-R formula under the rough ground surface conditions, Li et al. [15] have developed a three-dimensional (3-D) FDTD technique and validated the accuracy of the extended C-R formula for the fractal rough ground. Li et al. [16] employ a three-dimensional (3-D) FDTD method to analyze the lightning horizontal field over 2-D rough ground.

However, as the C-R approximate formula is in the frequency domain, it requires the computation of the field at several frequencies and the translation of the results to the time domain by a Fourier transform, and it may cause some errors. In 2008, Barbosa and Paulino [17] found that the C-R approximate formula can predict the initial part of the measured field wave from rocket-triggered lightning at close range (60m) with a satisfactory accuracy, however, the field wave predicted by the C-R approximate fades away much faster than the measured one. Also in 2008, Cooray [18] found the same behavior described by Barbosa and Paulino [17] by using Sommerfeld integrals, and Cooray [18] considered that the C-R approximate formula cannot be used in the vicinity of the channel.

In 2007, Barbosa and Paulino [19] developed an approximate time-domain formula which provides results in agreement with the C-R approximate formula. In 2008, Caligaris et al. [20] presented a time-domain version of the C-R approximate formula, which contains an integral that must be solved numerically. Barbosa and Paulino [21] extended their formula into the vicinity of the lightning channel and Barbosa and Paulino [22] further extended their time-domain formula (B-P approximate formula) into the horizontally stratified earth..

However, it is worth noting that, although Barbosa and Paulino [22] have validated the accuracy of their time-domain method for the stratified earth by using the frequency-domain formula presented by Wait [23], they only analyzed its accuracy at a constant distance of 300 m from the lightning channel, assuming that the conductivities of the two layer are

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0.01 S/m and 0.001 S/m. In their paper they considered the accuracy of their formula for layered earth surface and poor conducting earth is a subject to future investigation. In fact, we are more interested in the limits of the time-domain method, i.e., its accuracy at a distance of 50 m or 1000 m, in which distance range the lightning-induced voltage is interested. Whether the time-domain formula for stratified earth surface presented by Barbosa and Paulo[22] is valid for other distance ranges and different stratified earth has yet to be established. Therefore, in this paper, we will examine the accuracy of the time-domain method at distances ranging from 30 m to 1000 m from the lightning channel by using 2-D FDTD technique considering the horizontally stratified earth. Also, the C-R formula in frequency domain will be presented for comparison.

II. THE TIME-DOMAIN METHOD FOR HORIZONTAL FIELD CONSIDERING THE STRATIFIED EARTH

For the horizontally stratified ground as shown in Fig.1, the depth of the upper layer is \( h \), the lower layer is semiinfinite ground. The ground conductivity of the upper layer is \( \sigma_1 \), the dielectric constant \( \varepsilon_1 \), and the ground conductivity of the lower layer is \( \sigma_2 \), the dielectric constant \( \varepsilon_2 \).

According to the method presented by Barbosa and Paulo[22] (B-P approximate formula), the electric field at the surface of layered earth \( E_L(0,t) \) can be obtained from the electric field that would exist in a homogeneous earth with the electric parameters of the first layer by using a recursive equation [22], and the method is shown as below.

\[
E_L(0,t) = E_{Lt}(0,t) + 2 \sum_{i=1}^{n} k^i E_{Li}(2hi,t)u(t - 2ith) \\
E_{Lt}(0,t) = \int_0^t E_{lt}(0,t - \tau)H_L(2hi,\tau)d\tau \\
E_{Li}(0,t) = \int_0^t E_{lit}(0,t - \tau)H_L(2hi,\tau)d\tau
\]

Where \( \tau \) is the transit time from the surface to the boundary between the layers, i.e., \( \tau = h/v \) and \( v \) is the return stroke speed. \( E_{Lt}(0,t) \) is the field in a homogeneous earth with the electric parameters of the first layer \([19, 21]\). \( E_{Li}(2hi, t) \) is the reflective field arriving at the ground surface after the reflections times \( i \). \( \delta(t) \) is the Dirac’s Delta function. \( I_i \) is the modified Bessel function of first order. \( u(t) \) is the Heaviside’s function which is 1 for positive argument and 0 otherwise. \( k \) is the reflection coefficient in time domain as given below.

\[
k = \frac{\sqrt{\varepsilon_0} - \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_0} + \sqrt{\varepsilon_1}}
\]

Also, we will present the simulated results of the widely used method C-R formula in frequency domain for comparison. For the horizontal stratified ground, Shoory et al.[12] has given the expressions of C-R formula in frequency domain as below.

\[
E_L(0,d, jw) = -H_{d,n}(0,d, jw) \cdot W(0,d, jw) \cdot \Delta
\]

Where, \( E_{L}(0,d, jw) \) is the lightning horizontal electric field in frequency domain at the horizontally stratified earth surface, and \( H_{d,n}(0,d, jw) \) is the azimuth magnetic field at the perfectly conducting ground level. \( \Delta \) is the normalized surface impedance of stratified earth and \( W(0,d, jw) \) is the attenuation function corresponding to the propagation path [24, 25]. From equation (5), in order to obtain the azimuthal magnetic field on the finitely conducting ground, we firstly calculate the azimuthal magnetic field on the perfectly ground, and then consider the propagation effect by using the convolution integral as shown [13].

However, in B-P formulations, the lightning horizontal electric field at the homogeneously conducting earth surface can be calculated by equation (6)

\[
E_{H}(0,t) = \int_0^t E_{exp}(0,t - \tau)H_{exp}(0,\tau)d\tau
\]

Where, \( E_{H}(0,t) \) is the lightning horizontal electric field in time domain at the homogeneous earth surface. \( E_{exp}(0,t) \) is the horizontal electric field generated by an incident step magnetic field [21]. \( H_{exp}(0,t) \) is the azimuthal magnetic field produced by arbitrary lightning return stroke current and can be obtained as below.
\[ H_y(0, t) = \int_0^t H_{xy}(0, t - \tau) I(\tau) d\tau \]  

(7)

In equation (7), \( H_{xy}(0, t) \) is the azimuthal magnetic field produced by a step lightning current proposed by Rusck [26]. \( I(t) \) represents for arbitrary lightning return stroke current observed at the channel base.

III. VALIDATION OF THE TIME-DOMAIN METHOD BY USING 2-D FDTD

In this section, we will present the validation of the time-domain method by using 2-D FDTD method [27], [28]. The simulation domain of the FDTD technique is shown in Figure 1. The working space is 2000 m \( \times \) 2000 m, which is divided into square cells of 1m \( \times \) 1m, the time increment is set to 1.66 ns, and the first-order Mur absorbing boundary condition [29] is employed in order to simulate unbounded space. The parameters of the channel base current are those proposed by Rachidi et al. [30], and the transmission line return stroke model (TL) is employed, with a return stroke velocity equal to 150 m/\( \mu \)s. In this research, we present the validation of the time-domain method for the lightning subsequent return stroke current.

Fig.2 and Fig.3 show the accuracy validation of the B-P approximate formula by using our 2-D FDTD at the earth surface within distances ranging from 30 m to 1000 m from the lightning channel, considering the horizontally stratified ground referred to Case 1 and Case 2 in Table 1. We also present the results of the C-R approximate formula in frequency domain. In Fig. 2 and Fig.3, the bold line, dash line and dotted line correspond to 2-D FDTD, C-R approximate formula and B-P approximate formula, respectively. It is noted that the B-P approximate formula can approximately predict the field peak value with a satisfactory accuracy and the rise-time part of the field wave is similar to that of FDTD, however, in the vicinity of lightning channel (e.g., 30 m, see Fig.2 (a) and 3 (b)), the tail of the wave is much shorter than that by the 2-D FDTD method. Table 2 shows the statistics results of the field peak errors predicted by the time-domain method. From Table 2, its error is less than about 9\% for most cases. Generally, the C-R approximate formula in the frequency-domain predicts the similar field wave to that by the B-P approximate formula.

From the simulated results, note that, when the conductivity of the first layer is lower than the second layer, the corresponding field peak value is larger, however its half peak width is less. From equation (1), we can see that the horizontal electric field at the surface of layered earth \( E_h(0, t) \) is the sum of the electric field in a homogeneous earth with the electric parameters of the first layer and the reflective field from the boundary between the two layers. For the first layer with a lower conductivity (Case 1), the first reflective field is negative and due to its pushing downward predominantly, the falling edge of the total field waveform decreases sharply (see Fig.4 (a)). However, for the first layer with a larger conductivity (Case 2), all of the reflective fields are positive, the falling edge of its total field waveform decreases more slowly (see Fig.4 (b)). Therefore, for Case 1 and Case 2, the resultant field peak is determined by the homogeneous earth conductivity with the first layer and its falling edge depends on the contribution of the reflective field.
Figure 2. Comparison of the results predicted by the 2-D FDTD method, C-R formula and B-P formula for Case 1 at the ground surface for subsequent return stroke current parameters. Bold line, dash line and dotted line correspond to the results of 2-D FDTD, the C-R formula and B-P formula, respectively.

Figure 3. Comparison of the results predicted by the 2-D FDTD method, C-R formula and B-P formula for Case 2 at the ground surface for subsequent return stroke current parameters. Bold line, dash line and dotted line correspond to the results of 2-D FDTD, the C-R formula and B-P formula, respectively.

**TABLE I**
PARAMETERS FOR THE HORIZONTALLY STRATIFIED GROUND

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1$</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>$\varepsilon_1$</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>$h$ (m)</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

**TABLE II**
THE STATISTICS RESULTS OF THE PEAK VALUE ERRORS PREDICTED BY TIME-DOMAIN METHOD FOR SUBSEQUENT RETURN STROKE PARAMETERS
Table: Performance of FDTD method in predicting the electric field at different distances from the lightning channel

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Stratified earth</th>
<th>FDTD (V/m)</th>
<th>C-R (V/m)</th>
<th>B-P (V/m)</th>
<th>Errors of B-P (%)</th>
<th>Errors of C-R (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Case 1</td>
<td>2222.8</td>
<td>2378.5</td>
<td>2181.2</td>
<td>1.87% (41.6)</td>
<td>-6.99% (155.7)</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>891.6</td>
<td>898.6</td>
<td>848.2</td>
<td>4.87% (-43.4)</td>
<td>-10.79% (7.0)</td>
</tr>
<tr>
<td>50</td>
<td>Case 1</td>
<td>1288.6</td>
<td>1298.3</td>
<td>1211.1</td>
<td>6.01% (-77.5)</td>
<td>-0.75% (9.7)</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>497.9</td>
<td>483.7</td>
<td>454.3</td>
<td>8.76% (-43.6)</td>
<td>2.87% (-14.2)</td>
</tr>
<tr>
<td>100</td>
<td>Case 1</td>
<td>584.4</td>
<td>558.9</td>
<td>541.6</td>
<td>7.33% (42.8)</td>
<td>4.38% (25.5)</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>218.5</td>
<td>213.5</td>
<td>200.6</td>
<td>8.19% (-77.5)</td>
<td>2.28% (-5.0)</td>
</tr>
<tr>
<td>200</td>
<td>Case 1</td>
<td>260.7</td>
<td>244.1</td>
<td>246.9</td>
<td>5.29% (-12.8)</td>
<td>6.33% (-16.6)</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>96.3</td>
<td>93.1</td>
<td>90.1</td>
<td>6.44% (42.8)</td>
<td>3.30% (3.2)</td>
</tr>
<tr>
<td>500</td>
<td>Case 1</td>
<td>91.2</td>
<td>84.8</td>
<td>91.6</td>
<td>0.44% (9.4)</td>
<td>0.67% (6.8)</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>34.6</td>
<td>32.3</td>
<td>33.1</td>
<td>4.34% (-1.5)</td>
<td>6.45% (-2.3)</td>
</tr>
<tr>
<td>1000</td>
<td>Case 1</td>
<td>41.7</td>
<td>39.1</td>
<td>44.5</td>
<td>-8.11% (2.8)</td>
<td>3.33% (-2.8)</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>16.4</td>
<td>15.8</td>
<td>16.0</td>
<td>2.54% (-0.4)</td>
<td>2.96% (0.6)</td>
</tr>
</tbody>
</table>

Note: Values in brackets represent the relative error (%) and values outside brackets represent absolute error (V/m)

Figure 4. Generation of the electric field at the surface of layered earth at the distance 100 m from lightning channel for case1 (a) and case2 (b) for subsequent return stroke current parameters. (a) Bold line: resultant wave; thin line: homogeneous earth with $\sigma = \sigma' = 0.001$ S/m; dotted line: reflected waves from the second layer ($\sigma = 0$). (b) Bold line: resultant wave; thin line: homogeneous earth with $\sigma = \sigma' = 0.001$ S/m; dotted line: reflected waves from the second layer ($\sigma = 0$).

IV. CONCLUSION AND DISCUSSION

Barbosa and Paulino [22] have only validated the accuracy of their time-domain method (B-P formula) at the earth surface for the horizontally stratified earth at a constant distance 300 m from the lightning channel, assuming that the conductivities of the two layer are 0.01 S/m and 0.001 S/m. However, we are more interested in the limits of the time-domain method, such as its accuracy at other distance (e.g., 50 m or 1000 m from the lightning channel). Therefore, in this paper, we have examined the accuracy of the time-domain method at distances ranging from 30 m to 1000 m from the lightning channel considering the stratified earth by using 2-D FDTD technique and we also have presented the results of C-R formula in frequency domain for comparison.

Except for in the vicinity of channel (e.g., 30 m), it is noted that the B-P approximate formula can approximately predict the horizontal field peak values with a satisfactory accuracy within errors of 10% at distances from 50 m to 1000 m from the channel both for the subsequent return strokes. The rise-time part of the field wave predicted by the B-P approximate formula is similar to that predicted by 2-D FDTD, but the difference can be found at very close distances and in the decay times of the waves. For example, the tail of the wave predicted by the B-P approximation may be shorter than that predicted by the 2-D FDTD (e.g., 30 m and 50 m). Also, we find that the displacement current can be ignored and it has little effect on the accuracy of the B-P formula.

Comparison shows that the B-P formulation approximately have the same accuracy as the C-R formula, because both of them are based on the surface impedance concept, however, they are unlikely to provide very accurate results at extremely close range (e.g., 30 m), especially at late times, which has been investigated by Cooray [31].

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