A Method for Determining LDN Confidence Ellipse Information for Ground-Strike Points

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Abstract—This paper describes a method for determining confidence information for LDN ground-strike points. Ground-strike points can be determined using clustering algorithms and in the process, the strike point location is refined. However, the confidence ellipse information is lost when this is done. By treating stroke detections as multiple observations of the same event, Gaussian merging techniques can be used to develop a confidence ellipse for LDN ground-strike points.

Keywords-confidence ellipse; ground-strike point; LDN

I. INTRODUCTION

Modern Lightning Detection Networks (LDNs) are able to detect and resolve a position for multiple strokes within a single flash. This allows for multiple ground strike points to be determined for flashes with more than one termination points on the surface of the earth. There are a number of algorithms that have been developed to evaluate LDN data and determine the number and location of the ground strike points present in a flash. These algorithms have shown that it is also possible to determine a new location (and sometimes, a more accurate location) for each ground strike point from the stroke information [1, 2]. LDNs are also capable of evaluating their own location accuracy and is communicated in the form of error or confidence ellipses for each detected stroke [3]. However, when ground strike point algorithms are used, the resolved locations no longer maintain confidence information.

This paper proposes a method for developing confidence ellipses for ground-strike points determined from LDNs. The method is based on similar localization techniques used in robotics to locate objects.

II. BACKGROUND

A. Locating Lightning Strokes

While there are a number of different LDN (or Lightning Location Sensor (LLS)) manufacturers, the method of locating the geographic termination point of cloud-to-ground lightning events is generally the same. The sensors used in such networks detect the propagation of electromagnetic radiation (either the magnetic field or electric field component) caused by current flowing in the lightning channel. Triangulation techniques such as Magnetic Direction Finding (MDF) and Time-of-Arrival (TOA) are then used to determine the location at which a lightning event attached to the earth. Due to propagation effects and possible site errors, the sensors in a network do not agree exactly and optimization techniques such as the “least squares” method are used to determine the optimal geographic location [4].

B. Flashes and Ground-Strike Points

A single lightning event (ie. a channel attaching to a geographic location) may consist of multiple strokes and is known as a flash. Since LDNs detect the electromagnetic propagation of radiation as current flows through a channel, they are unable to distinguish lightning flashes but rather individual strokes. Given the slight errors that may occur, different strokes will often have different reported locations even if they all follow the same channel and terminate at the same location. Algorithms are used subsequently, to group a number of strokes into a flash.

It is also possible for a number of strokes in a flash to follow the same channel but then for the flash to branch and subsequent strokes to attach to a new ground strike point [1, 5]. For these reasons, there has been much discussion over the terms flash and stroke and it has been found necessary to introduce a third distinction, the ground-strike point [6]. Algorithms (based on k-means clustering and the geographical average of a number of strokes) are being implemented to distinguish groups of strokes that terminated at the same ground-strike point [2, 7].

C. Location Accuracy and Confidence Ellipses

Location accuracy refers to the difference in distance between the reported location of a lightning event by an LDN and the true location the lightning event attached to. Given the nature of the localization techniques used, it is possible for errors to occur based on the timing or angle information reported by individual sensors. For example, a sensor with a systematic error will continuously cause the location algorithm to misreport the location of a detected stroke. Even when systematic errors are not present, random errors can effect the reported location. To this end, LDN operators attempt to evaluate the location accuracy of a reported stroke [3, 4].
This is communicated in the form of an error or confidence ellipse – usually a major axis, minor axis and angle of rotation with the reported optimized location of the detected stroke located in the centre. The parameters used to determine this ellipse for each stroke are determined from observing the random errors of individual sensors for a large number of historic detections. For each past resolved stroke location, the difference between the optimized location (in terms of timing and angle information) and the time and angle information of each sensor that contributed to this optimized location, is calculated. This allows a distribution of errors to be determined for each individual sensor. If there are no systematic site errors, this distribution should be Gaussian in nature [3].

This means that a lightning stroke detected by an LDN can be represented as a two-dimensional Gaussian distribution (shown in canonical form in equation 1) where the optimized location is the mean $\bar{X}$ of the distribution and the parameters $\sigma_x$ and $\sigma_y$ and $\rho_{xy}$ are determined by merging the individual error distributions of each sensor.

$$p(X) = \frac{1}{2\pi\sqrt{|C|}} \exp \left( -\frac{1}{2} (X - \bar{X})^T C^{-1} (X - \bar{X}) \right) \tag{1}$$

where:

$$C = \begin{bmatrix} \sigma_x^2 & \rho_{xy} \sigma_x \sigma_y \\ \rho_{xy} \sigma_x \sigma_y & \sigma_y^2 \end{bmatrix}$$

From these parameters ($\sigma_x$, $\sigma_y$, and $\rho_{xy}$), it is possible to calculate a major axis, minor axis and angle of rotation for a confidence ellipse of any probability. Generally, the 50% confidence ellipse is provided.

**D. Object Localization in Robotics**

In the field of robotics, it is often necessary to be able to locate objects. As with the localization of lightning strokes, errors are present and similar confidence ellipses can be used to describe the location of objects. Work done by Stroupe et al. [8], involves improving the accuracy of an object localization by merging the error ellipses (or 2D Gaussian distributions) reported by two different robots observing the same object. The method simply involves multiplying the variance matrix of the observations of each robot to form a new variance matrix. Equation 2 describes how a new covariance matrix can be determined from the covariance matrices of two different observations. The new mean can then be found using equation 3.

$$C_{new} = C_1 - C_1 [C_1 + C_2]^{-1} C_1 \tag{2}$$

$$\bar{X}_{new} = \bar{X}_1 + C_1 [C_1 + C_2]^{-1} (\bar{X}_2 - \bar{X}_1) \tag{3}$$

**III. MERGING LDN STROKES**

While return strokes and the subsequent strokes that are detected by LDNs are not the same event, the channel and the ground termination point will be the same for each stroke (if they are all part of the same ground-strike point). In this way, these multiple strokes can be viewed as multiple observations of the same channel and location, similar to the object localization in section II.D.

Figure 1(a) shows two 2-Dimensional Gaussian distributions representing two stroke detections made by an LDN. If these two strokes were detected milliseconds apart, it is very likely they form part of the same flash and ground-strike point even though the mean of the distributions are not exactly the same. By applying equation 2, a new 2-Dimensional Gaussian can be created (figure 1(b)) to represent the flash or ground-strike point. It is also worth noting how the merged distribution has a much “tighter” variance or more confidence around the mean.

![Figure 1](image_url)

Figure 1. a.) Two 2Dimensional Gaussian distributions representing strokes detected by an LDN b.) the resulting merged 2Dimensional Gaussian distribution (adapted from [7]).

To apply this to real cases where ground-truth events are known and LDN reports are available, the following procedure is necessary:

- A flash grouping algorithm needs to be applied, to identify groups of strokes that form a flash.
- Time correlate grouped strokes to the ground-truth event.
- A ground-strike point algorithm needs to be applied to this subset of strokes to identify which strokes should be grouped to each ground-strike point.
• Equation 2 should be iteratively applied for each of these subsets to determine the covariance matrix for each ground-strike point.

• Equation 3 should be iteratively applied for each of these subsets to determine the mean of each ground-strike point.

Once the new mean and the covariance matrix are known, the new confidence ellipse parameters for the ground-strike points can be calculated in the same way as is done for strokes.

IV. CASE STUDY: BRIXTON TOWER

The Brixton tower is a tall tower (approximately 250 m) located in Johannesburg, South Africa (located at 26°11'32.82" South and 28°0'24.73" East). Lightning events attaching to the Brixton tower have been captured using a motion capture camera operating at approximately 30 frames per second over the last lightning season (October 2015 – February 2016). The camera faces east and has captured a number of flashes attaching to the Brixton tower. Figure 2 shows photographs of lightning attaching to the Brixton tower at 20:16:31 UTC, 20:45:41 UTC and 20:56:21 UTC on the 19 November 2015.

The South African Weather Service (SAWS) operates a lightning detection network in South Africa consisting of 24 Vaisala LS7000 sensors located across the country. The system was originally installed in 2005 and has thus been operational for 9 years. It has been show that the SALDN has an approximate median location accuracy of 250 m and 76% detection efficiency for upward lightning events (estimated to be higher for downward lightning events).

By applying the procedure in Section III, the SALDN detections of the events shown in figure 2 were identified. For each of the photographed flashes, four strokes were detected and grouped as a ground-strike point. As can be seen from the photographs, none of the events had multiple ground-strike points. Figure 3 shows the reported locations of the strokes relative to the Brixton tower as well as the geographic average (arithmetic average of latitudinal and longitudinal coordinates) of the four stroke locations. The 50% confidence ellipse for each stroke is shown too.

Figure 4 shows the four stroke locations for each event as well as the new mean of the “merged” ground-strike point. 50% confidence ellipses are shown for the strokes and the “merged” ground-strike point. As can be seen, the area of confidence for the ground-strike point is much smaller (but for the same confidence level) as for the individual strokes. Figure 5 shows the difference in location for the geographical average of the strokes for each event and the “merged” mean ground-strike point. The “merged” ground-strike point 50% confidence ellipse is shown too.

Figure 2. Lightning events attaching to Brixton tower on 19 November 2015 at a.) 20:16:31 UTC, b.) 20:45:41 UTC c.) 20:56:21 UTC.
Figure 3. Reported stroke locations (pink) and 50% confidence ellipses are shown for each of the photographed events in figure 2. The geographical average (red) is indicated.

Figure 4. Reported strokes locations (pink) and 50% confidence ellipses are shown for each of the photographed events in figure 2. The mean (blue) and 50% confidence ellipse of the “merged” ground-strike point is also shown.
Figure 5. The geographical average (red) and the mean (blue) and 50% confidence ellipse of the “merged” ground-strike point.

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<td>200.0</td>
<td>30.7</td>
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<td>109.4</td>
<td>500.0</td>
<td>12.3</td>
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Table 1 shows a comparison of the location errors relative to Brixton tower. For each event, the median location error of the four strokes and the median semi-major axis of the 50% confidence ellipse is calculated. As can be seen, both the geographical average and the “merged” ground-strike point mean result in smaller location errors than the individual stroke errors. Interestingly, the geographical average results in a smaller error than the “merged” ground-strike point. The semi-major axis of the “merged” ground-strike point is smaller than the median semi-major axis of the individual strokes which is to be as expected and indicates the more accurate location and the higher confidence in this location.

V. CONCLUSION

Since multiple strokes may attach to the same ground termination point, LDN stroke reports can be considered observations of the same ground-strike point. By applying object localization techniques used in robotics, it is possible to develop a confidence ellipse for LDN ground-strike points (or flashes). The “merged” confidence ellipse provides a more accurate mean and a greater confidence of the accuracy of this location.

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REFERENCES