Research of Lightning Leader Development in Mountainous Terrain Based on Conformal Mapping

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Abstract—The operating experience of power system suggests that the development of mountain lightning leader is certainly attracted by the mountain terrain for electric field distortion affection, resulting in increased CG lightning density in transmission line corridor area. This paper presents a simulation model applicable to lightning strike probability of transmission lines on abstract mountainous terrain. By a conformal mapping function, which has a similar shape as mountain, the abstracted 2D mountainous terrain will be mapped to the horizontal plane, then the image method is used to calculate electrical-potential field and Development Track of the lightning downward leader on mountainous terrain by LPM (leader progression model). The Striking Distance of EGM (electro-geometric model) is adopted as an important parameter to simply determine lightning strike point on lightning lead's last-jump. The simulation results show that the lightning distribution in some typical mountain terrain performs good agreement with the lightning location statistics in mountainous terrain area. Compared with the results calculated through EGM model, the shielding failure flashover probability of the edge phase of a 220 kV transmission line in Three Gorges area increases significantly through the new model, which considers the attraction effects on the lightning leader of mountain terrain. Therefore, the higher lighting flashover probability in mountain terrain can be well explained by the model.

Keywords- LPM; conformal mapping; image method; terrain; path of the lightning leader; lighting flashover probability

I. INTRODUCTION

Experience in electric transmission lines suggests that, under the same conditions the lightning flashover probability in mountain terrains is higher than in plains, implying that the specific terrain has a material effect on lightning outage rate of transmission lines. In China, the relevant Standards simply differentiate the mountainous terrain from the plain terrain, resulting in the shielding failure flashover probability calculated to be smaller than the actual experience. Based on the measured data, the classic Electro Geometric Model (EGM) [1,2] is proposed by Whitehead-Brown. The model was later improved by Eriksson [3]. After further improvement, a model was developed and recommended by IEEE. EGM has been widely used in engineering calculations. However, the results may be smaller than the actual experience when using EGM to calculate the shielding failure rate of transmission line in mountainous areas. The transmission lines' exposure distance calculated by EGM can reflect terrain parameters such as ground inclination, but it will cause the calculation error when lightning lead deflect for assuming the vertical drop of lightning. The moving track of lightning leader is certainly deflected by the mountain terrain for electric field distortion affection. Therefore, when the assumption condition is different from the EGM method, it will cause significant calculation error.

In addition, researchers of lightning proposed the Leader Progression Model based on the theory of long gap discharge. The Leader Progression Model (LPM) considers that the lightning leader always develops to the direction where the field strength is the greatest. A large number of research works have been carried out on the LPM. Chowdhuri[4] discussed the important parameters that affect the lightning shielding effectiveness of overhead power transmission lines, and gave a computation method in implementing the LPM. Jinliang He [5] built a LPM using the charge simulation method to calculate surface electrical field intensity of phase conductors and overhead ground wires, and proposed a method to calculate the shielding failure rate of transmission lines. Tavakoli and Vahidi[6-7] established a 3-D LPM, which considered some factors such as environment. However, none of them has considered the effect of the terrain. They only considered in the LPM for the plains. In recent years, some research has begun to consider the impact of terrain on the LPM. Bengang Wei [8] added mountainous to the LPM, where a skew line was taken to represent mountain terrain. Dellera and Garbagnati [14] depicted the path of the leader under different terrains. Hengxin He [9] developed a 2-D UHV transmission LPM based on Schwarz-Christoffel transformation where a mountain terrain was represented by a triangle. It is only an approximation of the actual terrain. In this paper, the moving track of lightning leader has been simulated under mountain terrain based on LPM, published by Dellera and Garbagnati in 1990. Combined with the EMG model which is recommended by IEEE to calculate the lighting flashover probability. The distribution of lightning is calculated in the Three Gorges Area's several typical hilly terrain. The result is compared with the lightning location statistics. The 30th tower in 220kV line in the Three Gorges Area is used as an example for the LPM simulation. The tripping caused by lightning stroke is depicted. The results

calculated by LPM and EGM recommended by IEEE are compared.

II.   LEADER DEVELOPMENT MODEL

Line charge is often used to simulate the downward leader in LPM. The downward leader develops step by step. This paper simulates influence of mountain terrain on the downward leader with LPM. During the process of the development of the downward leader, it produces induced voltage in objects on ground. If the induced voltage reaches a critical value, an upward leader is formed. Because the starting criterion of the upward leader, the development of the connection process and the connecting process of the downward leader is complex, and there is not a widely acceptable parameters of the upward leader, the LPM has been simplified in this paper without the upward leader progressmodeled.

The concept of "striking-distance" in EMG is used to describe the final jump process instead. Based on a large amount of observation data of lightning strike, the striking distance is practical in engineering. The charge density of the downward leader track in the LPM is of exponential distribution [10].

III.   THE CONFORMAL MAPPING OF TYPICAL 2-D TERRAIN

When LPM is used in a mountain terrain, the method of image cannot be directly used to make the image of the downward leader channel. Therefore, the mountain terrain must be transformed into a plain terrain where the image of the downward leader track can be captured. The electric potential can then be calculated and inversely transformed to the previous coordinate plane.

Conformal mapping has been widely used in electromagnetic field[11-13]. By using the character of conformal mapping function $w = f(z)$, the potential field in $z(x,y)$ plane with complicated boundary can be transformed into a potential field in $w(u,v)$ plane. Its edge can be simply drawn using the analytic function $w = f(z)$ and the solution on plane z can be acquired. The (x,y) is the coordination before the conformal mapping and the (u,v) is after.

The features of conformal mapping are:

1) If a function obeys Laplace equation before conformal mapping, the obedience will remain after the mapping.

2) If the conformal mapping function is $f(z)$, the absolute value of field strength of one spot on the plane w after conformal mapping is $|f'(z)|^{-1}$ times that of relevant spot on the plane z before conformal mapping.

3) Obtain the new potential function $\varphi(u,v)$ through conformal mapping and turn into the former independent variable using the analytic function $w = f(z)$, then come up with $\varphi(u(x,y),v(x,y))$, the former solution can be calculated by bringing in the value of x and y.

The conformal mapping functions for a hilltop terrain and a valley are [11]：

$$\begin{cases} w = \sqrt{z^2 + d^2}（\text{hilltop}） \\ w = \sqrt{z^2 - d^2}（\text{valley}） \end{cases} \qquad (1)$$

The hilltop and valley terrain can be transformed into plain terrain by conformal mapping function. where $z = x + i(y+e)$, z is the coordinate of a particular point in complex plane before the conformal mapping, $w = u + i(v+e)$. w is the coordinate of the point in complex plane after the conformal mapping. d and e are unknown parameters, being dependent on the specifics of a terrain.

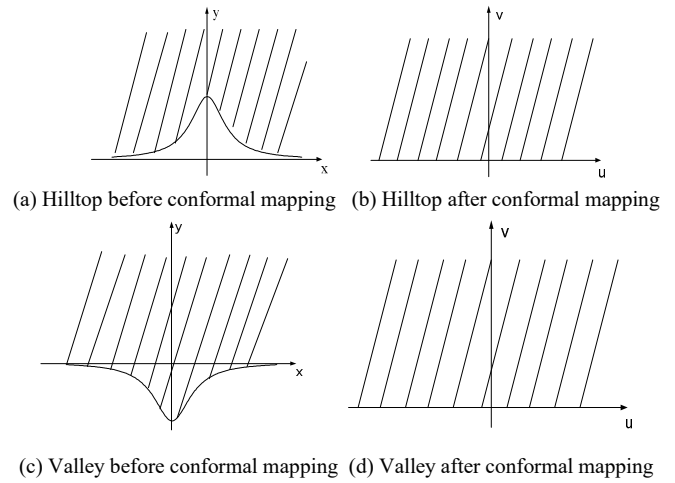The diagrammatic sketch of conformal mapping is shown in Figure 1.

(a) Hilltop before conformal mapping   (b) Hilltop after conformal mapping

(c) Valley before conformal mapping   (d) Valley after conformal mapping

Figure 1.   Schematic diagram of conformal mapping of hilltop and valley terrains

IV.   IMPROVED ELECTRO-GEOMETRIC MODEL

A.   Calculation of the Potential of the Downward Leader Head

In the schematic diagram of conformal mapping in Fig.2, a key step of the 2-D LPM when applied to a hilltop terrain. Assume that the leader path is a straight line, it will become a curve after conformal mapping.
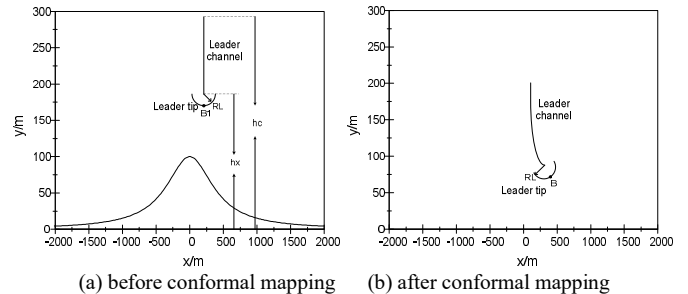

(a) before conformal mapping   (b) after conformal mapping

Figure 2.   Schematic diagram of a 2-D LPM

If the length of the leader path after conformal mapping being L, and the coordinate of the spot under study is $u(x,y)$ and $v(x,y)$, where x, y is the coordinate of the spot after conformal mapping, then there is:

$$dL = \sqrt{(u_y'(x,y))^2 + (v_y'(x,y))^2}\, dy \qquad (2)$$

After obtaining the image of the leader path using conformal mapping, the potential of any spot in this space is given in equation (3).

$$V = \int \frac{\rho(\xi)}{4\pi\varepsilon_0} \ln \frac{\sqrt{(u-u_0)^2 + (v+v_0)^2}}{\sqrt{(u-u_0)^2 + (v-v_0)^2}}\, dL \qquad (3)$$

If the leader channel before conformal mapping is represented as a straight line: $x = a + b \times y$, after the substitution of x and dL the potential of the spot B on the leader head being：

$$V_B = \int_{h_x}^{h_c} \frac{\rho(\xi)\sqrt{(u_y'(x,y))^2 + (v_y'(x,y))^2}}{4\pi\varepsilon_0} \ln \sqrt{\frac{r_1 + r_2}{r_1 + r_3}}\, dy \qquad (4)$$

Where

$$r_1 = (u(a+by,y) - u(a+by_{B_1}, y_{B_1}))^2$$
$$r_2 = (v(a+by,y) + v(a+by_{B_1}, y_{B_1}))^2 \qquad (5)$$
$$r_3 = (v(a+by,y) - v(a+by_{B_1}, y_{B_1}))^2$$

Where y_{B_1} is the y-axis of the leader head. Based on the features of conformal mapping, V_B is the potential of the spot B1 on the leader head before conformal mapping.
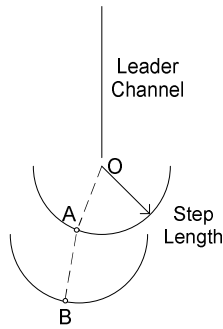
B. Judging the direction of Leader Progression

Figure 3. Schematic diagram of direction of lightning leader

The leader always develops to the direction where the field is the greatest. As shown in Figure 3, the procedure of determining the direction of leader progress is to draw a semi-circle with the dot O as the center, and the leader step as its radius. Then pick a few spots on the semi-circle and calculate the potential of these spots. Finally locate the spot whose potential difference is the greatest with leader head. Take spot A as an example, the OA is the direction of leader progression.

Draw a circle with A as the center, repeat the above calculations to obtain B. The direction of leader progression can be determined step-by-step in this way.

In EGM, it is believed that the lightning leader will not strike an object until it reaches the critical striking distance to the object. The striking-distance is the function of return stroke current:

$$S = 1.9 I_0^{0.9} \qquad (6)$$

Where S is the striking distance, in meters (m).

In this paper, the LPM is built without considering the progression of upward leader. The striking distance coefficient towards the earth in this paper is 0.93.

C. Analysis of Improved Simulation Model

It can be seen from the analysis of the model that many factors make influence to the model, such as leader channel charge density distribution, steepness of slope, slope position and so on. Then the factors are analyzed one by one.

1) Influence of Charge Density of Leader Channel on the Development Path of Leader: Three categories charge density of leader channel are compared, the constant distribution, linear distribution and exponential distribution. The charge density is a fixed value in constant distribution, however in propotion to the length of leader in linear distribution, then be exponent to the length of leader in exponential distribution. In the same terrain condition, same construct of tower and the same leader initial lateral distance, the deflection of the leader path is smaller under linear distribution than constant distribution. That is because under linear distribution, a point charge which can make some influence to the path is given to the head of leader. It is more reasonable to use the linear distribution than the constant distribution because the leader head have more electric charge usually. Under the same terrain condition, same construct of tower and the same lead initial lateral distance, the deflection of the lead path is larger under linear distribution than exponential distribution.
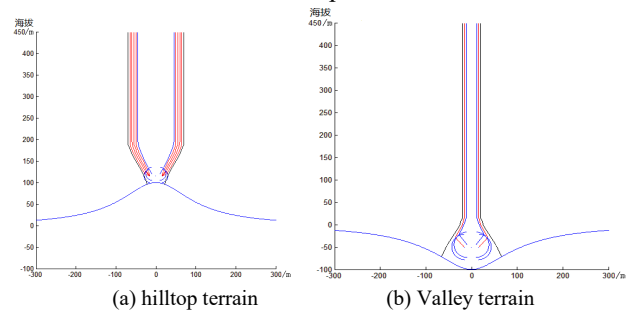
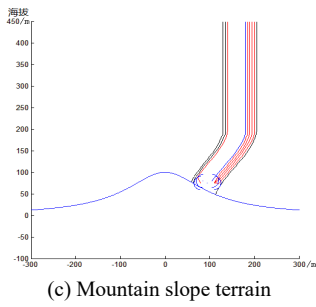| (a) hilltop terrain | (b) Valley terrain |

(c) Mountain slope terrain

Figure 4. Leaders development path in 3 typical terrain with 10 kA lightning current

2) Leader Development Path in Several Typical Terrain Condition. It can be seen from Figure 4(a) that the leader distortion is large under the influence of hilltop which leads to the increase of the exposure distance of the conductors, and increases the shielding failure rate of transmission lines. Instead, as it is shown in Figure 4(b) due to the enhancement of the shielding effect of the mountain slope to the wire, bare wire distance is very small, tower in the mountain slope terrain shielding failure rate is much smaller. While when the tower is in the mountain slope, the exposure distance of a wire close to the side of the mountain slope is much less than that of a wire that is far from the side of the hillside because of the shielding effect. And the wires that are far from the mountain slopes are much more likely to be shielding failure than that are around the side of the hill.

3) Comparison of Lightning Density in Typical Terrain Condition and Simulation Results. In this paper we selected 4 typical mountain terrain to get the relation between the simulation of lightning density and geographical condition .The elevation data extracted from typical terrain slope position factor, are simulated and compared with the lightning density, as is shown in Figure 5.
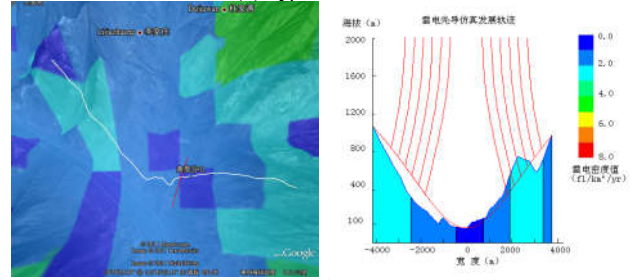
By comparing the typical terrain, the actual flash density is consistent with the simulation results.
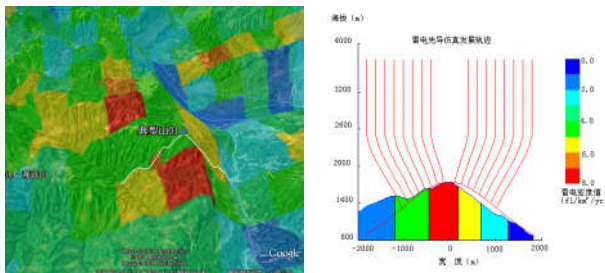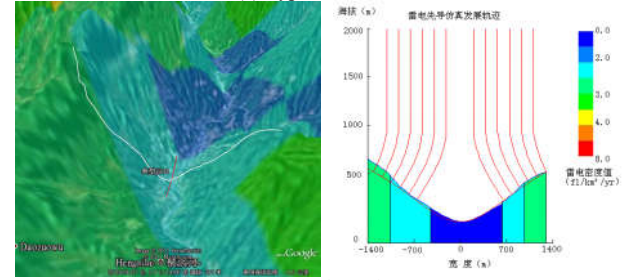

(a) typical terrain 1
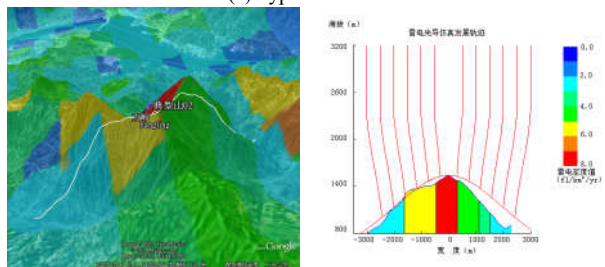
(b) Typical terrain 2

(c) Typical terrain 3

(d) Typical terrain 4

Figure 5. Comparison of 3D lightning density composite maps (left) with 2D simulation results (right) in 4 terrain

4) Analysis of Lightning Density under Different Slopes. When the lightning leader approached the ground, because of the influence of terrain the lightning leader will deflect, and this will lead to the lightning density's change. In this paper, the mountain terrain is divided into 3 different topographic position to analyze the lightning density under different slope positions. By calculate the ratio of the initial interval of lightning leader in the air L1 and the interval of lightning leader on the ground L2 under different topographic positions, the variation of the density of drop lightning is reflected. The ratio of L1 and L2 is defined as lightning density coefficient. To a certain extent, lightning density coefficient can reflect the variation of the lightning density. Here the definition of the angle between the tangent line and the horizontal line of the height corresponding to the height of the mountain is the mountain slope. When the mountain slope is 30 degree, the simulation results of lightning density in different slope under mountain terrain are shown in Table 1.

TABLE I. SIMULATION RESULTS OF THE LIGHTNING DENSITY AT DIFFERENT TOPOGRAPHIC POSITION IN PEAK TERRAIN (SLOPE =30)

	No.1 Topographic Position	No.2 Topographic Position	No.4 Topographic Position
initial interval of lightning leader in the air/m	88	99	1005
interval of lightning leader on the ground/m	33	40	1000
lightning density coefficient（initial interval of lightning leader in the air / interval of lightning leader on the ground）	2.67	1.57	0.95

The calculation results show that, there are only the topographic position 1, 2 and 4, no 3 when the slope is 30

degrees. The number 2 topographic position has the greatest ratio of initial interval of lightning leader in the air and the interval of lightning leader on the ground, 2.67, which means in this area the lightning density coefficient becomes greater under the influence of the hilltop terrain. While the ratio of number 4 topographic position is 0.95, which means in this area the lightning density coefficient becomes smaller. The number 3 topographic position's is 1.57, little greater.

Comparison of simulation results with statistical results in peak terrain, the lightning density and flash density coefficient show a downward trend, indicating that the simulation results and statistical results are consistent to a certain extent.

With increasing of slope, numerical difference between the lightning density and flash density coefficient is becoming greater, but the overall trend is basically consistent with. This is because the idealized model proposed in this paper to simplify the influence of other factors in the actual flash density.

*5) Influence of Mountain Slope on the Development of Lightning Leader.*The development of lightning leader is certainly attracted by the mountain terrain for electric field distortion affection, and the leader is always in the direction of the largest field strength. Therefore, the steepness of the mountain will have an impact on the leader's development path. The leader's deflection will become greater when the steepness of the mountain is becoming larger. It is believed that the deflection of leader will increase with the increase of the steepness of the mountain. Because the increase of the steepness of the mountain can enhance the effect of the space electric field's distortion, which can enhance the influence of the leader's development.

The relation between the slope lightning density coefficient and the changes of steepness is with the steepness' increase the lightning density coefficient increased. This is because the steeper the mountain, the more intense effect on the space electric field, resulting in the deflection of the lightning leader more serious, which makes the lightning density coefficient increases with the steepness of the increases.

D. Analysis of Simulation Results

(a) Flashover tower (b) The cross section of the terrain
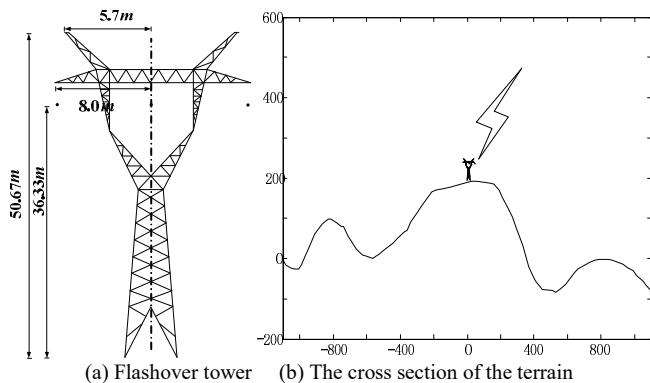
Figure 6.   30th typical 220kV tower

A 30th tower in 220 kV line which is on the top of a mountain in Three Gorges Area, the west of Hubei, has been used as an example to calculate. 2005-07-21 19:03, flashover occurred to the 30th tower's C (right) phase insulators, and there were obvious sighs of discharge on wires and insulators. Lightning location system shows that the lightning current amplitude was 36.5kA. Judged by the Flashover site environment, traces of discharge and lightning location system's query results, it is concluded that the lightning accident for the thunder and lightning around the wire is because of the strike. Flashover tower's structure and the cross section of the terrain are shown in Fig.6.

Through the 30th tower in the line corridors vertical section of the terrain extracted through Google Earth and the type (6)'s hilltop function, it can be fitting to the terrain, in which the solid line for the actual terrain, dots and dashes for through preserving transform angle function to fit the terrain. Figure.7 (a) shows the path of the downward leader when the lightning current amplitude is 36.5kA (only the right side of the tower is shown). Figure.7 (b) shows the path of the downward leader's enlarged view.
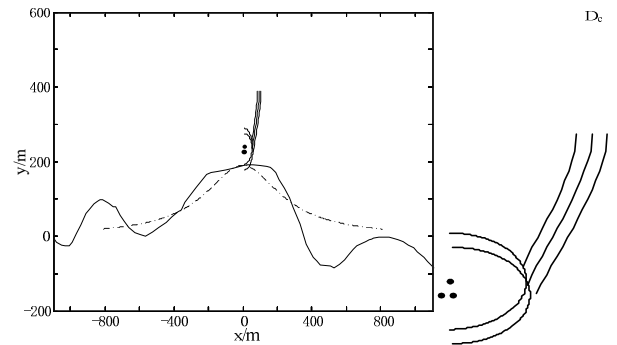
(a)path of the downward leader (b) the downward leader's enlarged view

Figure 7.   Path of downward lightning leader when the lightning current amplitude is 36.5 kA

With the same terrain and tower parameters, the exposure distance calculated using the EGM recommended by IEEE-1997, under different lightning current amplitude, are compared with the result of using LPM, as shown in Table.2.

TABLE II.   COMPARISON OF EXPOSED DISTANCE THAT CALCULATED BY LPM AND EGM UNDER HILLTOP TERRAIN

lightning current amplitude/kA		15.5	25	30	36.5
exposed distance of edge phase conductor Dc/m	LPM	8	9	5	6
	EGM	2.62	2.35	2.31	2.3

The comparison shows that, with the same lightning current amplitude under hilltop terrain, the exposure distance calculated by LPM is greater than that calculated by EGM. At lightning current amplitude of 36.5 kA the exposure distance calculated by LPM is 6.0 m, while that calculated by EGM is 2.3 m.

The greater the exposure distance, the greater the probability that the wire is subjected to the lightning shielding failure, which means the probability of the winding of the edge of the transmission line is increased obviously considering the attraction of the peak terrain to the lightning guide.

V. CONCLUSIONS

1) In this paper, a lightning strike probability model for transmission lines is proposed, based on a special kind of conformal mapping function, mapping the 2D mountainous terrain to the horizontal plane to make it more convenient to calculate with image method.

2) Based on the maximum direction of the electric field, the 2D leader development model is built to simulate the development of lightning in mountain terrain area. The model of the "striking-distance" in electro-geometric model is simplified to "last jump".

3) The calculation results application this method in the Three Gorges area of several typical mountain terrain of the distribution of lightning have a good correspondence with the statistics of lightning location data. Taking a 220 kV line hilltop lightning tower,which is in the mountainous area of the Three Gorges as an example, comparisons with the IEEE Recommended EGM calculation of edge phase conductor exposure distance show that under the same lightning current amplitude and terrain conditions, the exposure distance calculated by this method is greater, indicating that transmission line edge phase conductor shielding failure probability is larger after considering the hilltop terrain of lightning leader attraction. That can explain why the lighting flashover probability is higher in mountain terrain better.

ACKNOWLEDGMENT

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