Discussion of Current Dependent Grounding Resistance using an Equivalent Circuit Considering Frequency-dependent Soil Parameters

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***Abstract*— This paper discusses an equivalent circuit to consider the frequency dependence of soil parameters. The paper describes a calculation method of the constants of the equivalent circuit. Calculated results using the equivalent circuit considering the constants agree well with experimental results. The equivalent circuit can consider the frequency dependence and the current dependence of grounding resistance simultaneously. Thus, the equivalent circuit is very useful for lightning surge analysis. An influence of the frequency dependence of soil parameters on the current dependence of grounding resistance is discussed using the equivalent circuit.**

Keywords-frequency-dependet soil parameter; soil ionization; equivalent circui

I. INTRODUCTION

The grounding resistance is one of the most fundamental factors to determine voltages and currents. It is very important for the human safety and the insulation design for instruments and power apparatuses to make the voltages low by the reduction of the grounding resistance. The grounding resistance is expressed by the functions of the configuration and dimension of a grounding electrode, and soil resistivity. The grounding resistance is not constant for high-frequency current such as lightning current, and shows time dependence as illustrated in Figs. 1 and 2 (inductive, flat and capacitive variations). Thus, the grounding resistance should be treated as impedance [1], namely grounding impedance. The grounding impedance shows capacitive variation for high grounding resistance. On the other hand, that varies as inductive impedance for low steady-state value.

The frequency dependence of soil parameters is one of the factors affecting the grounding impedance [2, 3]. The frequency-dependent parameters are estimated on the basis of measured results. A number of experimental results are required to validate the frequency-dependent soil parameters. This paper discusses the determination of constants of an equivalent circuit of grounding resistance to consider the frequency dependence of soil parameters.

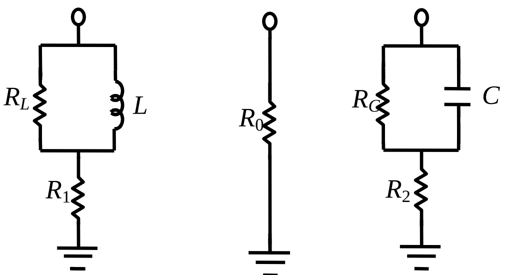


Figure 1 Equivalent circuits of grounding impedance

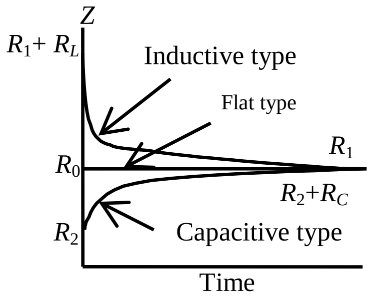


Figure 2 Transient response of grounding impedance

II. FREQUENCY DEPENDENCE OF GROUNDING IMPEDANCE

A. *Lightning Surge Characteristics of Grounding Resistance*

The grounding resistance shows the following characteristics for lightning currents having steep front and high amplitude.

(1) time dependence (transient phenomena for constant circuit elements)
(2) frequency dependence
(3) current dependence

This chapter briefly explains these characteristics.

(1) Time dependence of grounding resistance

A grounding conductor of more than several meters should be treated as a distributed-parameter line as shown in Fig. 3, because surge velocity on a grounding conductor is about one-third of velocity of light in free space. Sending end voltage V_s of a grounding conductor for $2nT \leqq t < 2(n+1)T$ (T is the traveling time of the conductor) [4] is given by

$$V_s = E + P_V E\left[\sum_{k=1}^{n} Q_s{}^{k-1} Q_r{}^{k} e^{-2k\theta l} + S_n(x-nl) + \sum_{k=0}^{n-1} S_k(l)\right] \tag{1}$$

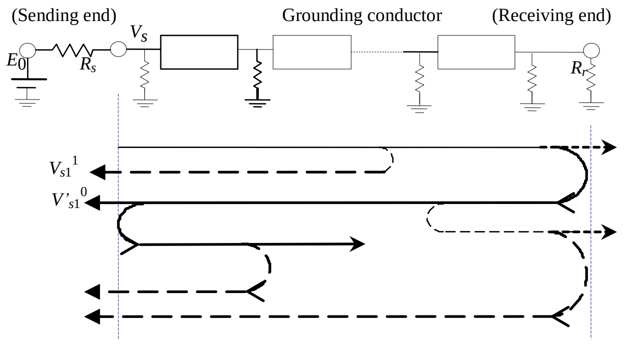


Figure 3 Traveling voltages on a grounding conductor

$$S_k(x) = F(x)e^{-2k\theta l}\left\{(k+1)(Q_sQ_r)^k + k(Q_sQ_r)^{k-1}Q_r{}^2\right\} \qquad (2)$$

$$F(x) = -\frac{1}{2}\left(1-e^{-2\theta x}\right) \qquad (3)$$

where $x = vt/2$, v is the surge velocity of the grounding conductor, P_s is the refraction coefficient of voltage at the sending end, Q_s is the reflecction coefficient of voltage at the sending end, Q_r is the reflecction coefficient of voltage at the receiving end, E is the applied voltage to the grounding conductor, l is the conductor length, θ=$Z_0G/2$, G is the conductance per length of the grounding conductor, Z_0 is the surge impedance of the grounding conductor.

The transient response of grounding conductor as shown in Figs. 1 and 2 can be explained by the relation between the grounding resistance and the surge impedance using eq. (1).

Thus, the voltage on a grounding electrode varies as a function of time by the time dependence even if the electrical circuit elements are constant.

(2) Frequency dependence of grounding resistance

Line impedance is dependent on the frequency due to the skin effect of the finite conductivity of the soil and the line [5]. Switching surge analysis considers the frequency dependence [6]. The resistivity to cause the skin effect is assumed to be constant. On the other hands, the soil resistivity and the soil permittivity depend on the frequency. These frequency dependences must be distinguished for discussions. The skin effect affects the frequency dependence of line impedance. The frequency dependence of the soil parameters affects the line admittance. When both the frequency dependences are discussed simultaneously, the frequency dependences are treated as the frequency dependence of impedance, which is caused by the skin effect, and the frequency dependence of admittance due to the frequency dependence of soil parameters. Off course, the frequency dependence of the soil resistivity affects the line impedance. This paper does not consider the frequency dependence of the impedance.

(3) Current dependence of grounding resistance

The grounding resistance decreases as injected current increases. This phenomenon is caused by soil ionization around a grounding electrode [7]. Wherever current density branching off through an electrode exceeds a critical value, the soil ionization occurs. The ionized zone with low resistivity grows with the increase of the injected current. As a result, the current dependence of the grounding resistance is observed. The higher the soil resistivity, the hevier the current dependence of the grounding resistance. Smaller grounding electrode shows more reduction of the grounding resistance [8].

B. Transient Response of Grounding Resistance

A small grounding electrode shows a transient characteristic given by a resistance R_a and a capacitance C_a as illustrated in Fig. 4 based on the electromagnetic field theory. The R_a and C_a are given by

$$R_a=\rho K \qquad (4)$$

$$C_a=\varepsilon K^{-1} \qquad (5)$$

$$R_aC_a=\rho\varepsilon=\tau \qquad (6)$$

where K is determined by the dimension and configuration of the grounding electrode, ρ is the soil resistivity, ε is the soil permittivity, ε is given by $\varepsilon_r\varepsilon_0$, where ε_r is the relative soil permittivity, and ε_0=8.854x10^{-12}.

For example, coefficient K for a driven rod is expressed using Dwight's formula as follows [9]:

$$K = \frac{1}{2\pi l}\ln\frac{2l}{r} \qquad (7)$$

Time constant of grounding impedance is proportional to the soil resistivity and the soil permittivity, and is independent of the dimension and configuration.

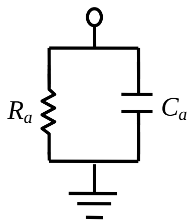


Figure 4 Circuit of grounding resistance based on electromagnetic theory

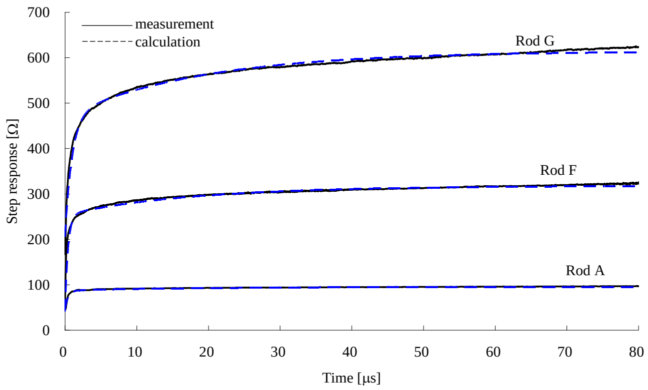
C. Frequency-dependent Soil Parameters

Grounding impedance of a small grounding electrode often shows capacitive variation. This phenomenon is mainly caused by the frequency dependence of soil parameters. Visacro and Alipio proposed simple formulas for the frequency dependence of relative soil resistivity ρ_r and soil relative soil permittivity ε_r [3]. ρ_r is given by ρ/ρ_0, where ρ_0 is the soil resistivity for low frequency. These parameters are given by

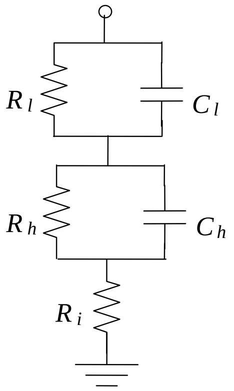
$$\rho_r(f) = [1+1.2\times10^{-6}\rho_0{}^{0.73}(f-100)^{0.65}]^{-1} \qquad (8)$$

$$\varepsilon_r(f) = 7.6\times10^3 f^{-0.4}+1.3 \qquad (9)$$

These parameters are decreased as the frequency becomes higher.



(a) Step response



(b) Equivalent circuit
Figure 4 Grounding resistance for low current.

TABLE 1 MEASURED STEADY-STATE GROUNDING RESISTANCE

Electrode	Rod A	Rod F	Rod G
Soil resistivity	143 Ωm	495 Ωm	949 Ωm
Resistance	92 Ω	318 Ω	610 Ω

TABLE 2 CONSTANTS OF THE EQUIVALENT CIRCUIT OF RODS

	R_l [Ω]	C_l [μF]	R_s [Ω]	C_s [nF]	R_0 [Ω]
Rod A	8.0	2.3	42.0	11.0	45.0
Rod F	63.0	0.28	213	2.0	41.0
Rod G	143	0.14	266	3.7	205

III. MEASUREMENT RESULTS OF GROUNDING RESISTANCE OF ROD ELECTRODE

A. Measurement Results of Step Response of Grounding Resistance of Rod Electrode

Fig. 4 (a) shows step response of grounding impedance of a rod electrode [9]. The step response is estimated from experimental results for stepwise applied current using a numerical Laplace transform [10]. The experiments are carried out in three yards with different soil resistivity. The soil resistivity and grounding resistance for low frequency are shown in Table I. The steady-state grounding resistance is measured using fall of potential method. The soil resistivity is estimated from the steady-state grounding resistance using Eqs. (4) and (7). The rod electrode has the same dimension of r=7 mm and l=1.5 m.

B. Equivalent Circuit of Grounding Impedance

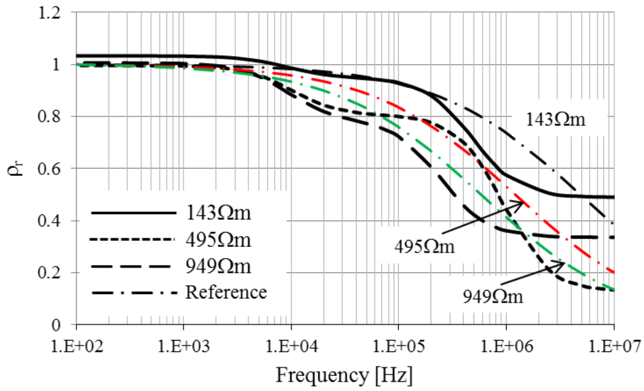
Fig. 4 (b) shows an equivalent circuit of the grounding impedance based on the step response in Fig. 4(a) [11]. R_l-C_l and R_h-C_h parallel circuits represent low and high frequency components, respectively. Grounding electrode has surge impedance, and shows an initial value for stepwise current. R_i represents the value for steep-front current. R_l+R_h+R_i corresponds to the steady-state grounding resistance. The step response of the equivalent circuit is expressed by

$$Z_t(t) = R_i + R_l\left[1-\exp\left(-\frac{t}{C_l R_l}\right)\right] + R_h\left[1-\exp\left(-\frac{t}{C_h R_h}\right)\right]. \qquad (10)$$

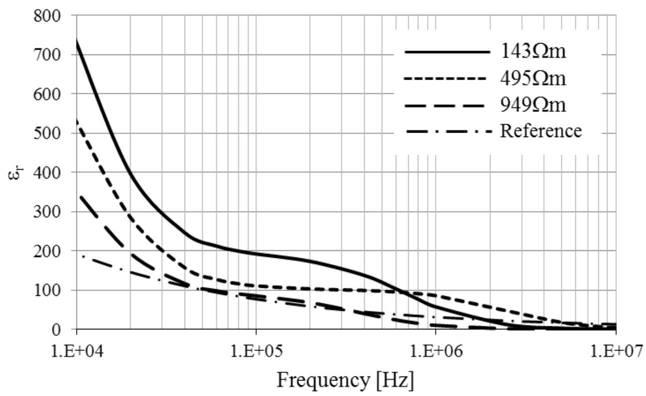
Table II shows the values in the equivalent circuit. Fig. 4(a) includes calculated results of the step response using the equivalent circuit. The equivalent circuit has sufficient accuracy in comparison with the experimental results.

IV. FREQUENCY DEPENDENCE OF GROUNDING IMPEDANCE

Fig. 6 shows calculated results of frequency dependence of relative soil resistivity ρ_r and relative soil permittivity ε_r using



(a) Relative soil resistivity



(b) Relative soil permittivity
Figure 6 Frequency dependence of soil parameters estimated using equivalent circuit

the equivalent circuit. Calculated results of the relative soil resistivity and relative soil permittivity using Eqs. (8) and (9) are included in Fig. 5. Frequency range in Fig. 5 is the same as that in Ref. [3]. The relative soil permittivity is high in low-frequency region. The low-frequency grounding impedance acts grounding resistance. Therefore, the error of the relative soil permittivity does not significantly affects the step response.

The time constant of the R_l-C_l parallel circuit of the equivalent circuit for low frequency discussed in this paper is about 20 μs. Thus, the relative soil permittivity for less than 50 kHz might include large error. Measured results of surge velocity on grounding conductors are approximately $v_0/4$ to $v_0/3$, where v_0 is the velocity of light in free space. Therefore, the relative soil permittivity must take about 10 to 20. The relative soil resistivity and relative soil permittivity for more than 1 MHz might show large error. Thus, the formulas (8) and (9) satisfactorily agree with the measured results obtained using the equivalent circuit.s

V. Determination of Constants of Equivalent Circuit

The equivalent circuit is very convenient because it can be easily used in transient analysis tool such as the EMTP [12], and frequency dependence and nonlinear phenomena can be considered simultaneously. The nonlinear phenomena of grounding resistance due to the soil ionization is one the nonlinear phenomena related to the grounding resistance.

(1) *Ri*

When stepwise current is injected into a grounding electrode of more than a few meters, traveling voltage appears. Therefore, the initial grounding impedance is not zero. This phenomenon is regarded to be steady state after some reflections on the grounding electrode. The voltage at t=2T is estimated using eq. (1). The voltage is given by

$$V_s = E + P_V E[Q_r e^{-2\theta l} + S_{k=1}(l) + S_{k=0}(l)] \qquad (11)$$

Considering grounding electrode is opened at the receiving end, and current in measurement is injected through high resistance, Qs=Qr=1, and Ps=2. Considering

$$2\theta\lambda = Z_0/P_0 \qquad (12)$$

$$E = Z_0 I_s, \qquad (13)$$

The voltage is given by

$$V_s / I_s = 3Z_0 (e^{-2\theta l})^2 \qquad (14)$$

Eq. (17) is adopted to represent distributed parameter line as R_i.

(2) R_h-C_h

Considering rise time of lightning currents is several micro-seconds, ρ_r and ε_r at 100 kHz are used. Time constant τ_h is given by the following equation using these constants.

$$\tau_h = \rho_r \varepsilon_r \rho_0 \varepsilon_0 \qquad (15)$$

The resistance R_h is given by

$$R_h = (R_0 \rho_r / \rho_0 - R_i) \qquad (16)$$

The capacitance C_h is given by

$$C_h = \tau_h / R_h \qquad (17)$$

(3) R_l-C_l

The circuit for low frequency is estimated at 100 Hz because Eq. (8) is derived on the basis of fundamental low-frequency soil resistivity at 100 Hz. R_l-C_l are obtained using the variables at 100 Hz as follows:

$$\tau_\lambda = \varepsilon_\rho \rho_0 \varepsilon_0 \qquad (18)$$

$$R_l = (R_0 - R_i - R_h) \qquad (19)$$

$$C_l = \tau_l / R_l \qquad (20)$$

Fig, 7 illustrates an approximation of the frequency dependence which is approximated by stair wise characteristic.

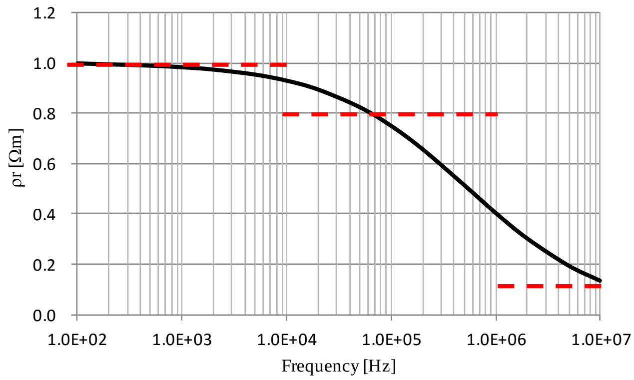


Figure 7 Approximation of frequency dependence for determination of constants of the equivalent circuit

Table 3 is obtained by the proposed method. The surge impedance Z_0 of the grounding electrode is 75 Ω [13] to estimate R_i. Fig. 8 shows a comparison of calculated results of step response of the grounding resistance using the equivalent circuit and the constants in Table 3 with the measurement results in Fig. 5(a).

TABLE 3 CONSTANTS OF THE EQUIVALENT CIRCUIT OF RODS

	R_l [Ω]	C_l [μF]	R_h [Ω]	C_h [nF]	R_i [Ω]
Rod A	6.81	0.23	41.09	2.20	44.1
Rod F	52.47	0.10	125.1	2.26	140.4
Rod G	147.0	0.069	287.0	1.72	176.0

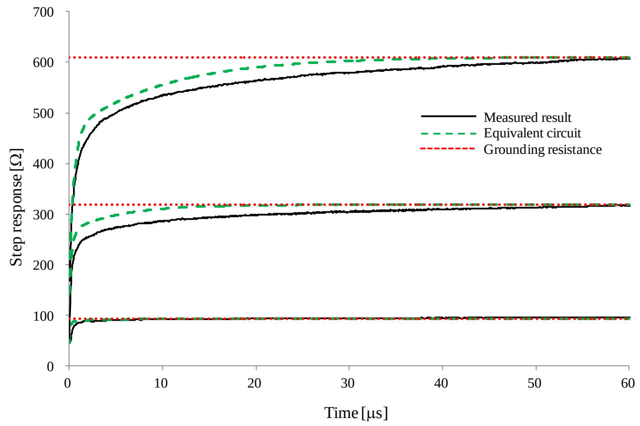


Figure 8 Comparison of calculated results using constants of equivalent circuit with measured results.

From Fig. 8, the calculated results relatively agree with the measurement results. The difference between the calculated and measurement results is mainly caused by the difference of the soil permittivity from Fig. 6(b). Fig. 9 shows calculated results using capacitances having twice higher values.

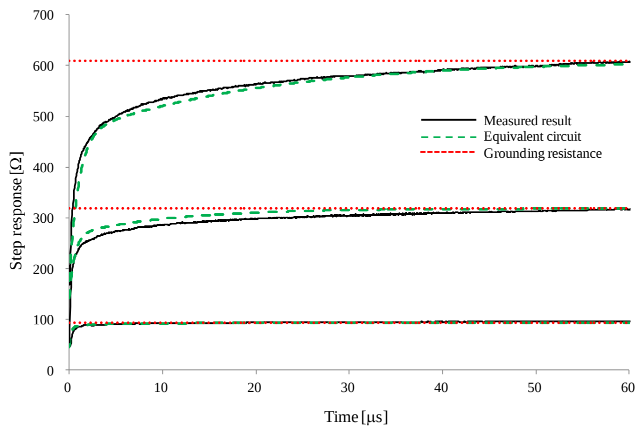


Figure 9 Comparison of calculated results using improved constants of equivalent circuit with measured results.

It is clear from Fig. 9, the higher capacitance gives better results. The velocity propagating on a grounding conductor is about $v_0/4$ to $v_0/3$ from experimental results. As a result, higher soil permittivity is acceptable. Thus, the equivalent circuit can represent the frequency dependence of grounding resistance by the proposed method. As is well known water content and temperature affect the soil parameters. Therefore, the further measurements and study of the frequency dependence are necessary.

VI. CURRENT DEPENDENCE OF GROUNDING RESISTANCE USING THE EQUIVALENT CIRCUIT

A. Measurement Results of Current-dependent Grounding Resistance

The soil ionization is one of the most important factors related to the grounding for lightning surge analysis. The grounding resistance is decreased due to the increase of equivalent radius of grounding electrode caused by the soil ionization as the injected current becomes higher. The following definition of grounding resistance $R_s(I_m)$ is adopted in this paper to estimate the current dependence of the grounding resistance.

$$R_s(I_m) = \frac{V_m}{I_m} \qquad (21)$$

where V_m is the crest value of applied voltage, I_m is the crest value of applied current

Measured results of the current dependence of the grounding resistance of the rod are shown in Fig. 10 [14]. A low impulse current generator (LIG) circuit is different from a high impulse current generator (HIG) circuit by the existence of a damping resistor [15].

Fig. 10 indicates the current dependence is affected by the applied current waveform. The frequency dependence of the grounding resistance is one of the reasons for the waveform dependence. Therefore, the frequency dependence should be considered for accurate lightning surge analysis.

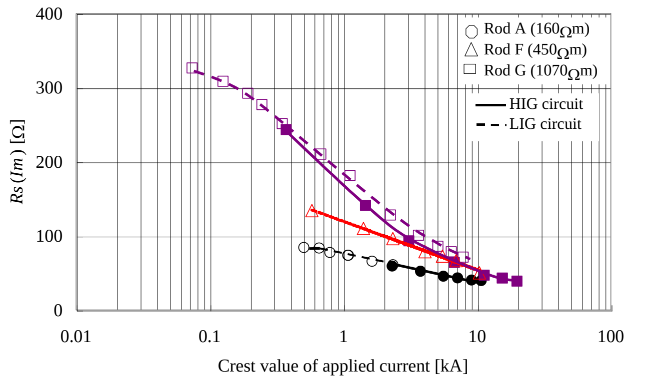


Figure 10 Measured results of current-dependent grounding resistance of a rod electrode.

B. Comparison of Simulation using the Equivalent Circuit with Measurement

This paper discusses the influence of the frequency dependence of the grounding impedance on the current dependence. The contour of soil ionization zone is estimated using the Liew-Darveniza model [16]. The resistivity of the soil ionization zone is not considered for simplicity. The critical electric field to develop the soil ionization zone of 400 kV/m is used in this paper [17, 18]. The grounding resistance under de-ionization process is assumed to be the same as the lowest value. Fig. 11 shows simulation results of the current dependence of the rods A, F, and G using the equivalent circuit. Waveform of injected current is T_f/40 μs. Fig. 12 shows calculated results of voltage waveforms on the grounding electrodes. T_f in Fig. 12 is 1 and 10 μs.

From the calculated results, the influence of the frequency dependece on the current dependence of the grounding resistance cleraly occurs for low currents and high grounding resistance.

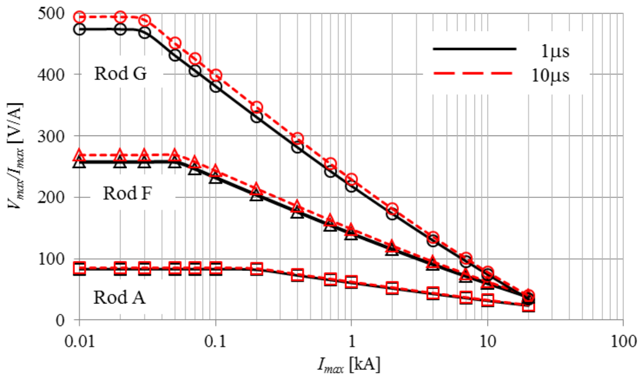


Figure 11. Calculated results of current-dependent grounding resistance of a rod electrode considering the frequncy dependence.

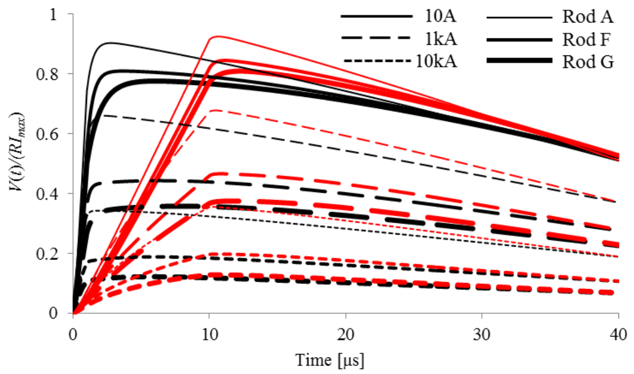


Figure 12 Calculated results of voltage waveform at the current injected point.

VII. Conclusion

This paper proposes an equivalent circuit consisting of *R-C* parallel circuits to represent the frequency dependence of grounding resistance, and a method to determine the constants of the equivalent circuit. The calculated results relatively satisfactorily agree with the measurement results. The equivalent circuit can be used in the EMTP, and is applicable to the frequency dependence and the current dependence of the grounding resistance due to the soil ionization simultaneously.

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