



# A Modified Pi-Shaped Circuit-Based Model of Grounding Electrodes

P. Yutthagowith

Faculty of Engineering,  
King Mongkut's Institute of Technology Ladkrabang  
Bangkok, Thailand  
kypeeraw@kmitl.ac.th

**Abstract**— This paper proposes a modified pi-shaped circuit-based model of grounding electrodes. The validity length and soil resistivity of the proposed model are from 2 m to 40 m and from 50  $\Omega\text{m}$  to 2000  $\Omega\text{m}$ , respectively. The equivalent circuits in the form of pi configuration are modeled by lumped parameters composing of inductance, resistances, and capacitances. The voltage responses to the standard lightning currents of grounding electrodes either calculated by an electromagnetic model or from experiments are matched with those of the proposed model. A curve fitting technique is employed to extract parameters of the equivalent circuit. The satisfactory agreements have been observed. The simple soil ionization model can readily be adapted in the proposed model. The proposed model is very useful in implementation with a circuit simulator such as EMTP, Pspice, and so on. The proposed circuit model can be adapted for being the models for development and design of the grounding system for lightning protection system in transmission and distribution systems in an effective way.

**Keywords**- grounding electrodes; equivalent circuit; electrical transient; lightning current; modified pi model; soil ionization

## I. INTRODUCTION

The grounding system is used for current dissipation from lightning and faults in a power system. In addition, it is also used as a reference in electrical circuits of a power system and electronic. Conventionally, the simple grounding systems for transmission towers and distribution poles are horizontal and vertical electrodes buried in soil. Therefore, the design of the grounding electrode in the power system is necessary for reducing the over-voltage and preventing damage to equipment and human life.

Predictive calculation of ground potential rise on a grounding system due to lightning in the transmission and distribution systems is most essential for an economical insulation design of the transmission and distribution systems. Conventionally, circuit-theory based methods and transmission line approaches assuming a transverse electromagnetic (TEM) propagation mode were employed for the calculation of the over-voltages due to a lightning strike or faults in the transmission and distribution systems. The

assumption of the TEM propagation mode might not be correct for the grounding system. This behavior cannot be expressed by a time-independent model unless the rise time of the current is much longer than the round-trip time of the travelling wave. To obtain more accurate results, full-wave approaches or electromagnetic models based on the method of moment (MoM) [1], [2], the finite-difference time-domain (FDTD) technique [3], [4], and a hybrid electromagnetic-circuit methods such as hybrid electromagnetic model (HEM) [5] and the partial element equivalent circuit (PEEC) method [6] are more appropriate than the circuit-theory based and transmission line approaches. However, the electromagnetic models require large amount of memory and long calculation time for analyses of transient phenomena. Therefore, they are not appropriate for analyses of transient phenomena in a large system. The circuit-theory based method is still an effective approach for analyses of over voltage in the real system. Thus, it will be advantageous if the equivalent circuit of a grounding electrode having high accuracy comparable with the electromagnetic models is available for adapting in a circuit-theory based simulator for transient analyses.

A number of circuit modeling approaches of grounding electrodes have been proposed for numerical simulations. The equivalent circuits are represented either by lumped-parameter or distributed parameter circuits [7]. Time domain and frequency domain approaches [5], [6] are available to develop a circuit model of the grounding system. The time domain approach is based on an observation of a time response. In the frequency domain approach, the frequency characteristic of grounding impedance is obtained by taking the ratio of numerically evaluated frequency responses of measured or calculated voltages and currents. The parameters of inductance ( $L$ ), resistance ( $R$ ), and capacitance ( $C$ ) are then extracted from the frequency characteristic of the impedance.

In this paper, a circuit-based model in the form of modified pi configuration is proposed. Downhill simplex method [1] was employed to extract the circuit parameters of simple grounding electrodes from the voltage responses to standard lightning currents. The validity ranges of soil resistivity and electrode lengths are from 50  $\Omega\text{m}$  to 2000  $\Omega\text{m}$  and from 2 m to 40 m,

respectively. The voltage responses were calculated by the PEEC method and matched to those of the proposed model. The frequency dependent soil parameters are neglected. The proposed equivalent circuit is quite effective for analyses of the transient behavior of simple grounding electrodes used for a grounding system of transmission towers and distribution poles.

## II. CIRCUIT-BASED MODEL OF GROUNDING ELECTRODES

In transmission and distribution systems, the design of a grounding electrode for preventing over voltage in a low frequency range is well known and can be studied from many literatures and standards. The primary interesting characteristic of the grounding electrode is its input impedance or impedance to remote neutral ground. Traditionally at low frequencies, this impedance is represented by a single resistor and at higher frequencies by conventional lumped R-L-C circuits as shown in Fig. 1(a) and (b). The modified pi-shaped circuit in Fig. 1(c) is proposed and investigated in this paper.

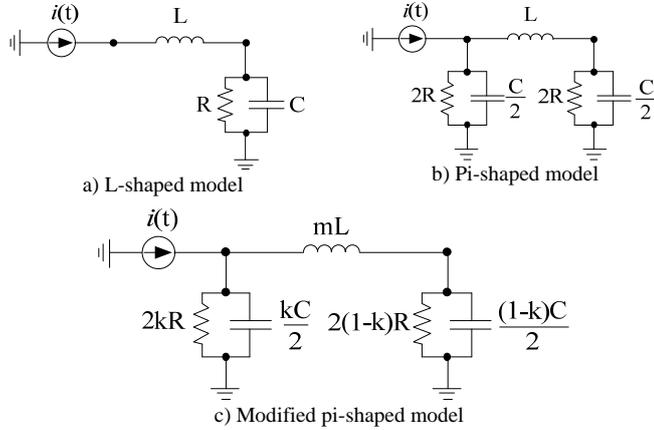


Fig. 1. Equivalent circuit of a grounding electrode.

The parameters of a horizontal grounding electrode,  $R$ ,  $L$ , and  $C$  of the equivalent circuit can also be derived and calculated as follow equations [8];

$$R(l, a) = \frac{\rho}{\pi l} \left[ \log \frac{2l}{\sqrt{2ad}} - 1 \right] \quad (\Omega), \quad (1)$$

$$C(l, a) = \pi \epsilon l \left[ \log \frac{2l}{\sqrt{2ad}} - 1 \right]^{-1} \quad (\text{F}), \quad (2)$$

$$L(l, a) = \frac{\mu l}{\pi} \left[ \log \frac{2l}{\sqrt{2ad}} - 1 \right] \quad (\text{H}). \quad (3)$$

where,  $l$ ,  $a$ , and  $d$  are length, radius, and buried depth of a grounding electrode, respectively.  $\rho$ ,  $\epsilon$ , and  $\mu$  are resistivity, permittivity, and permeability of soil, respectively.

However, behavior of a grounding electrode in the higher frequency range of lightning currents being upto 10 MHz (in case of subsequence stroke lightning current), and especially in transient state is quite different from that at the low

frequency range. Fig. 2 [9] shows the normalized standard lightning current waveforms.

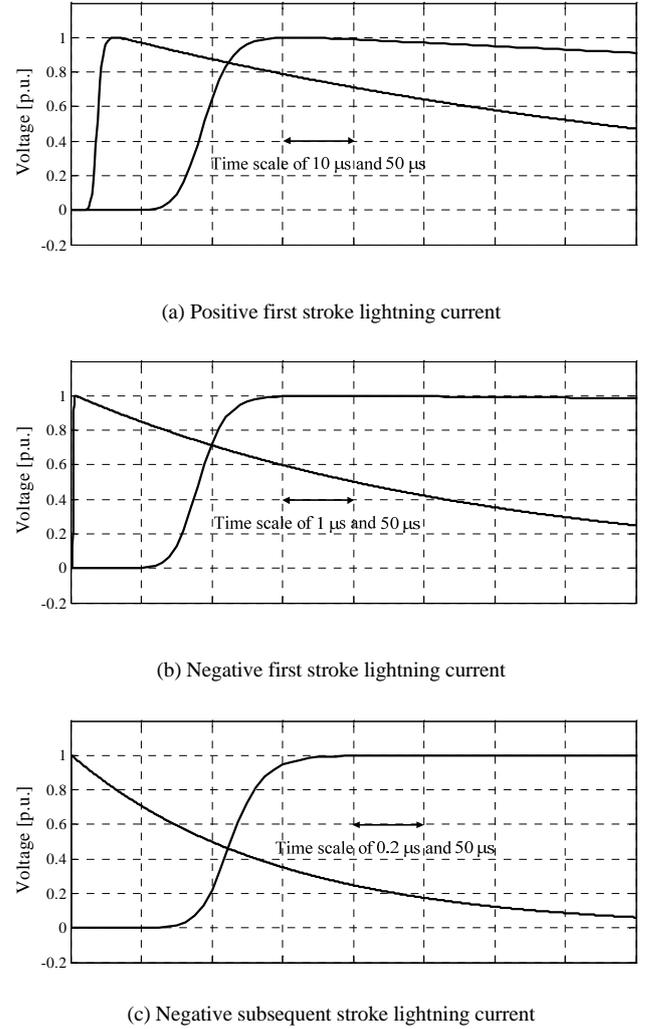


Fig. 2. Normalized waveforms of the standard lightning currents.

## III. LIMITATION OF THE CONVENTIONAL CIRCUIT-BASED MODEL OF A GROUNDING ELECTRODE

As illustrated in Fig. 1, the equivalent circuit of a grounding electrode is taken into account effects of inductance and capacitance of the grounding electrode. However, the conventional circuit-based models are not accurate. The limitation of the conventional circuit-based model is expressed by considering the voltage responses of a 20-m long grounding electrode buried in the soil with soil resistivity of 100  $\Omega\text{m}$  and with the buried depth of 1 m as shown in Fig. 3. It is found that L-shaped model provides the over estimation of the voltage response, and the pi-shaped model provides the under estimation when they are compared with the response of the electromagnetic model. The black solid line stands for the response of the electromagnetic model. To obtain the accurate response, the ratios of  $k$  and  $m$  in Fig. 1(c) are necessary to adjust. Good agreement of the voltage response between the

electromagnetic and modified pi-shaped models is found in case of  $k$  and  $m$  equaling to 0.23 and 0.28, respectively.

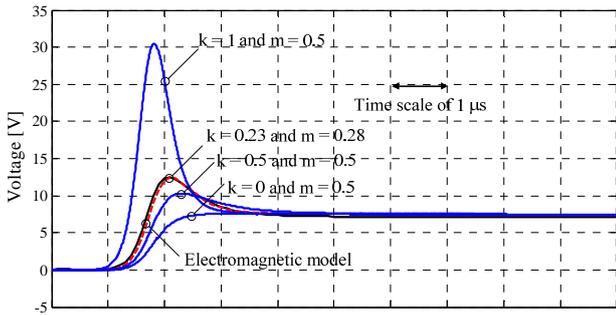


Fig. 3. Voltage responses to the negative first stroke lightning current.

#### IV. MODIFIED PI-SHAPED CIRCUIT-BASED MODEL OF A GROUNDING ELECTRODE

In this paper, the modified pi-shaped circuit as shown in Fig. 1(c) is selected to be a proposed model of a grounding electrode. The circuit parameters, i.e. resistance ( $R$ ), capacitance ( $C$ ), and inductance ( $L$ ) are calculated by eqs. (1) to (3). As mentioned in the previous section, it is necessary to adjust the values of  $k$  and  $m$  in the proposed model. Downhill simplex method is employed to obtain the responses of either the electromagnetic model or experimental results.

##### A. Verification of the Proposed Model with Experimental Results

To confirm the validity of the proposed model, the experiment on voltage response of a grounding electrode [10] is considered. The configuration of the grounding electrode and soil parameters are given in Fig. 4.

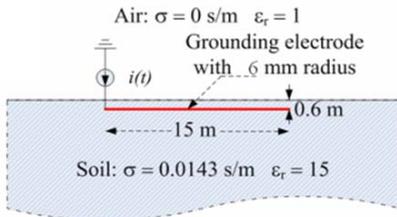


Fig. 4. Soil parameters and configuration of the grounding electrode in the experiment.

As shown in Fig. 5, good agreement between the voltage responses of the electromagnetic and proposed models is observed. The calculated values of  $k$  and  $m$  are 0.23 and 0.29, respectively.

The soil ionization behavior can readily be included with the proposed model. In this paper, according to Cigre [11], the soil ionization can be represented by the nonlinear resistance as eqs. (4) and (5).

$$R_i = \frac{R}{\sqrt{1 + \frac{i(t)}{I_g}}} \quad (\text{H}), \quad (4)$$

$$I_g = \frac{E_c \rho}{2\pi R^2} \quad (\text{H}), \quad (5)$$

where,  $E_c$  is critical electric field strength of soil. Normally,  $E_c$  is set to 200 kV/m to 400 kV/m.

To verify the representation of the soil ionization with the proposed model, the experiment [12] on a horizontal grounding electrode with radius of 4 mm, length of 5 m, and buried depth of 0.6 m is considered. In the simulation by the proposed model, soil resistivity and relative permittivity were set to be 42  $\Omega\text{m}$  and 10, respectively. Good agreement of experimental and computed ground potential rise (GPR) is observed as shown in Fig. 6.

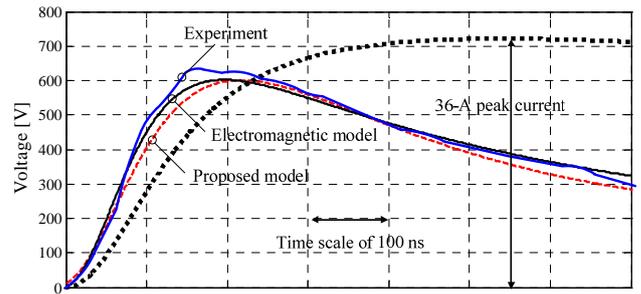


Fig. 5. Voltage responses to the injected current from experiment and from calculation by the electromagnetic and proposed models.

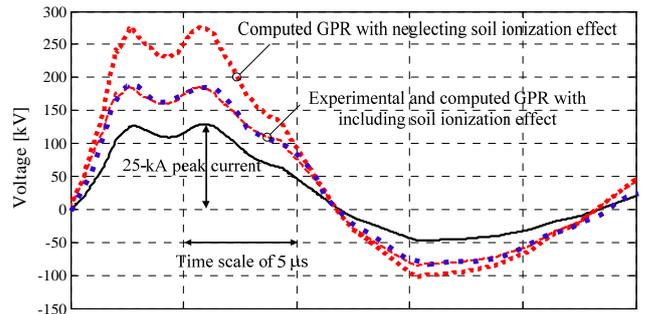
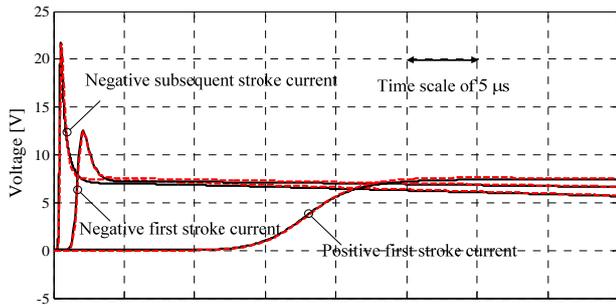


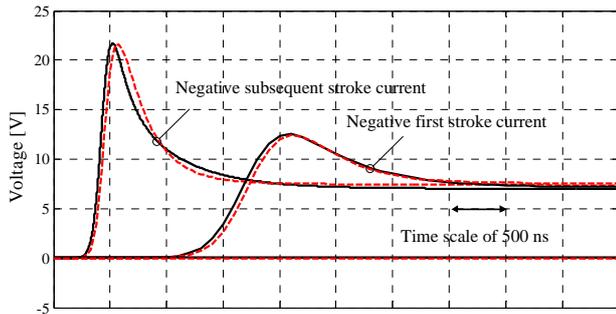
Fig. 6. Voltage responses to the injected current from experiment and from calculation by the proposed model.

##### B. Verification of the Proposed Model with an Electromagnetic Model

The proposed model was verified with an electromagnetic model in cases of grounding electrodes with various lengths from 2 m to 40 m and with soil resistivity from 50  $\Omega\text{m}$  to 2000  $\Omega\text{m}$ . The buried depth and relative permittivity of soil were set to be 1 m and 10, respectively. The normalized standard lightning currents as shown in Fig. 2 were injected at the receiving end of a grounding electrode. The voltage responses from the proposed model were calculated and matched to those from the electromagnetic model. Satisfactory agreements are observed in all cases. Examples of the voltage responses are shown in Figs. 7 and 8.

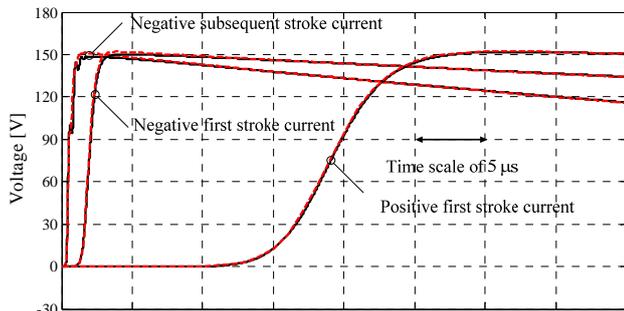


(a) Long time scale

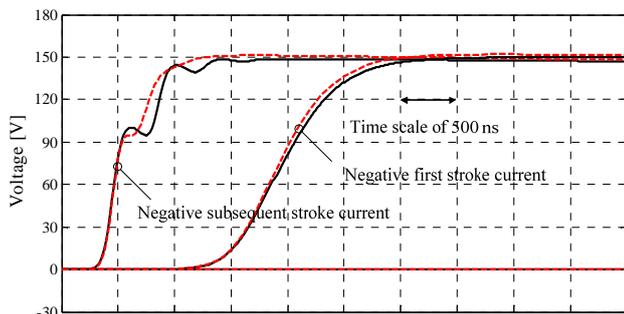


(b) short time scale

Fig. 7. Comparison of voltage responses calculated by the electromagnetic and proposed models in a case of a 20-m length grounding electrode and soil resistivity of 100  $\Omega\text{m}$ .



(a) Long time scale



(b) short time scale

Fig. 8. Comparison of voltage responses calculated by the electromagnetic and proposed models in a case of a 20-m length grounding electrode and soil resistivity of 2000  $\Omega\text{m}$ .

Black solid and red dashed lines stand for results calculated by an electromagnetic model and proposed model, respectively.

## V. CONCLUSIONS

In this paper, a modified pi-shaped model of the grounding electrode was proposed. The circuit parameters are determined by matching voltage response of the proposed model to that either of an electromagnetic model or from experiment. The proposed model was verified with experimental results and an electromagnetic model in cases of grounding electrodes with various lengths from 2 m to 40 m and with soil resistivity from 50  $\Omega\text{m}$  to 2000  $\Omega\text{m}$ . The effect of soil ionization is represented by non-linear resistance which is readily adapted to the proposed model. Good agreements between the voltage responses from proposed model and from either an electromagnetic model or experiment are found in all considered cases. The proposed model is very useful in implementation with a circuit simulator such as EMTP, Pspice, and so on. The proposed circuit model can be adapted for being the models for development and design of the grounding system for lightning protection system in transmission and distribution systems in an effective way.

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