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# Space Charge Distribution Inside the Corona Sheath During a Return Stroke

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*Abstract*—**A new form of the space charge distribution inside the channel corona sheath that represents a generalization of previously adopted or calculated space charge densities is numerically analyzed. On one hand, the use of appropriate parameter values produces the previously established corona charge densities as special cases of this generalized distribution. On the other hand, the influence of the new space charge distribution on the corona sheath dynamics can be examined. The simulations showed that the new generalized function possesses excellent features regarding flexibility in modeling of different corona sheath charge distribution. Besides, it proved to be simple and easy to calculate. Although it is applied in the generalized traveling current source model in this study, it could also be readily implemented in transmission-line type return stroke models.**

***Keywords-lightning; corona sheath model; return stroke model; corona space charge density.***

## I. Introduction

Electric field created by the charge located at the thin and lengthy conducting channel core is predominantly radially directed and surpasses the breakdown value. Repulsive electrostatic forces through the streamers push the charge out from the channel core, until the radial electric field drops below the breakdown value. The electrostatic field has a dominant influence on charge transfer in the corona sheath.

Consequently, the leader channel comprises a thin core and a corona sheath which forms radially around it. 2 MV/m magnitude of the breakdown field was assumed in [1], while in [2] adopted the value of 1 MV/m.

The majority of the leader charge is located within the corona sheath radius, which is on the order of meters, while the highly conductive channel core, with a diameter assessed at around one centimeter, in effect transports the whole of the axial current, [3, 4].

Traveling-current-source-type of models represent the return stroke process as a current source that propagates upwards, injecting the current pulse on its way into the channel. Each section of the channel corona sheath itself produces certain current pulse as a product of neutralization of the corona sheath, positioned around the central core of the leader channel.

The generalized lightning traveling current source return stroke model (GTCS) [5] couples the channel-base current and the initial line charge density in the corona sheath. It introduces the so called channel discharge function or the channel charging function (the corona sheath is discharged by the positive charge coming from the channel core) [7]. The implementation of the GTCS model in the corona sheath models enables the study of the dynamics of the corona sheath.

Model of corona sheath that surrounds the thin core of the lightning channel has been investigated by using a new space charge function. The corona sheath model [6-10] divides the sheath into two charge zones: zone 1 (surrounding the channel core with net positive charge) and zone 2 (surrounding zone 1 with negative charge). We introduced a new expression for charge distribution inside both zones that represents a generalization of previously used space charge distributions. Using appropriate parameter values, previously examined corona charge densities are obtained as special cases of this generalized distribution. Furthermore, the influence of the new distribution on the corona sheath dynamics is examined. Expressions that describe how the corona sheath radius evolves during the return stroke are obtained from the adopted corona sheath model and the new charge distribution. The new distribution can also be readily applied to transmission-line type models.

## II. Space Charge Distribution Inside Corona Sheath

The generalized form of the radial space charge density distribution in zones 1 and 2 has recently been introduced as [11]

$$\rho = \rho_0 \exp(-r/\lambda)/r^{m(r)}, \qquad (1)$$

where $\rho_0$ is the normalizing space charge constant ( $\rho_0^+$ for the positive charge in zone 1 and $\rho_0^-$ for the negative charge in zone 2), $r$ is radial distance from the channel core axis, $\lambda$ and $m(r)$ are the corona sheath parameters. The space charge density (1) is valid for the whole corona sheath, including the channel core. In principle, the corona sheath parameters differ for positive charge (zone 1) and for negative charge (zones 1 and 2). However, for simplicity and clarity of the numerical simulations we used the same corona sheath parameter values for both zones. This form of the radial charge distribution is convenient for the representation of different physical conditions inside the corona sheath prior or during the return stroke. If, for example, the model of the corona sheath with constant space charge density is used [6-8], it follows from (1) that $\lambda \to \infty$ and $m = 0$. If the corona model assumes constant electric field in the sheath, the space charge is inversely proportional to radial distance [9, 10]. In that case, it follows from (1) that $\lambda \to \infty$ and $m = 1$. Other distributions needed for the study of the corona sheath dynamics can be obtained by an appropriate choice of the parameter values in (1).

Values of $\rho_0$ in (1) are obtained in [11] through normalization of the total charge in each zone. This yields

$$\rho_0^+ = q_0^+ f^+ / (2\pi \cdot I_1), \quad \rho_0^- = -q_0^+ / (2\pi \cdot I_2), \qquad (2)$$

where

$$I_1 = \int_0^{R_{out}^+} \frac{\exp(-r/\lambda)}{r^{m-1}} dr \ , \quad I_2 = \int_0^{R_{out}^-} \frac{\exp(-r/\lambda)}{r^{m-1}} dr \ . \qquad (3)$$

If the numerical package containing the subroutine for the lower incomplete gamma function $\gamma(s,x)$ is used, (3) can be expressed as

$$I = \lambda^{2-m}\,\gamma(2-m, R/\lambda), \ \gamma(s,x) = \int_0^x t^{s-1}\exp(-t)\,dt. \qquad (4)$$

## III. Corona Sheath Radius During Return Stroke

The time dependence of the outer boundary of zone 1 during return stroke depends on the model of the corona sheath. Two different sub-models for the channel sheath are proposed in [7]: one with exponential charge decay in zone 2 ( $R_{out}^- = const$ ), and another with a decrease of zone 2 ( $R_{out}^- \neq const$ ). In this study we used the model with the decrease of zone 2. The radius of zone 1 is derived in [11]

$$R_{out}^+ = \frac{2B}{\xi}\left( f^+ - \frac{I_1(R_{out}^+)}{I_2(R_{out}^-)} \right), \qquad (5)$$

where $\xi = E_r^+ / \left|E_r^-\right|$, $B = q_0^+ / (4\pi\varepsilon_0 \left|E_r^-\right|)$. The velocity of the outer boundary of zone 1 is

$$v_{out}^+ = \frac{dR_{out}^+}{du} = \frac{2B}{\xi}\left( \frac{df^+}{du} - \frac{d}{du}\left(\frac{I_1}{I_2}\right)\right), \qquad (6)$$

where $u = t - z/v$ is the normalized time, $z$ is the channel height, $v$ is the return stroke speed.

## IV. Numerical Procedure

The unknown variable $R_{out}^+$ from (5) that contains integrals $I_1$ and $I_2$ can be obtained by using two different numerical approaches. Other variables in (5) are known from the analysis of the channel sheath dynamics within the GTCS model [8]. The initial line charge density is known from the electric field measurements in the immediate vicinity of the lightning channel [13]

$$q_0^+ = -2\pi\varepsilon_0 r E(0), \qquad (7)$$

where $E(0)$ is the initial field magnitude at the beginning of the return stroke stage (starting point on the trailing edge of the recorded field waveform [13]). Breakdown field magnitudes on the outer surfaces of zones 1 and 2 are $E_r^+$ and $\left|E_r^-\right|$, respectively. Their values are adopted from the literature, and they are in the range of 1-2 MV/m, [1, 2, 6]. For the applied model of the corona sheath with shrinkage [7-10] the radius of zone 2 is

$$R_{out}^- = q_0^+(1 - f^+) / (2\pi\varepsilon_0 \left|E_r^-\right|), \qquad (8)$$

where the channel charging function $f^+$ is taken from [8]. An example of the set of the initial channel sheath parameters is given in Table I, based on the measurements in [13].

TABLE I. Initial Channel Sheath Parameters [10, 13]

| **Flash S0033** | *Peak electric field* [MV/m] | $q_0^+$ [μC/m] (7) | *Initial channel sheath radius* $R_{out}^-$ [cm] | *Breakdown electric field at* $R_{out}^-$, $\left|E_r^-\right|$ [MV/m] |
|---|---|---|---|---|
| Stroke 1 | 1.168 | 6.67 | 6 | 2 |

As stated above, two numerical approaches should be applied to solve (5). One starts with the rearranging of (5) to express the integral $I_1$ in the form

$$I_1(R_{out}^+) = I_2(R_{out}^-)\left[ f^+ - R_{out}^+ \xi / (2B) \right] \qquad (9)$$

As the first step, the initial value for $R_{out}^+$ is accepted. It could be zero but other positive initial values are also acceptable. Substituting it into the right hand side of (9) one obtains the value $I_1$ on the left hand side of (9).

If $I_1 > 0$ from (9) the new value of $R_{out}^+$ is obtained by use of the inverse lower incomplete gamma function. This iterative procedure repeats until some final value for $R_{out}^+$ is reached.

However, this algorithm is not successful in every step of the iteration process. Depending on the time instant and the values of the input parameters for charge distribution $\lambda$ and $m$ in(1), some steps of the iterative procedure give $I_1 < 0$ for a certain value of $R^+_{out}$. Since in that case the inverse lower incomplete gamma function does not exist, to overcome the problem (5) is used instead of (9) for calculating a new value of $R^+_{out}$. In the iterative procedure, as stated above, $R^+_{out}$ continues to be calculated by returning to (9) .

## V. RESULTS

In order to demonstrate the possibilities of the generalized formula for the channel space charge density (1), we examined the different charge distributions as well as the dynamics of the corona sheath, which corresponds to different physical models of the discharge. The results are shown in Fig.1-7.

Solving (5) or (9) with the help of (8), the radii of zones 1 and 2 vs time are depicted in Fig.1 for adopted magnitudes of the breakdown fields $E^+_r = 1$ MV/m and $\left|E^-_r\right| = 2$ MV/m . The corona model with the shrinkage of zone 2 is used [7-10], but in this case, the space charge distribution differs which can be clearly seen in Fig.2. The space charge distribution is depicted in the instant of time $t \cong 1.5$ μs where the maximum of the radius of zone 1 occurs. Sharp discontinuities of the space charge densities are visible at the outer surface of zone 1 in accordance with the model of the adopted corona sheath. It should be noted that, in this case, the charge density equals zero in the channel core (due to its high conductivity), which is more realistic assumption than the assumption of the constant space charge density [6, 8]. Net positive space charge density in zone 1 in Fig.2 does not exceed 2 *m*C/m. The time discharge constant of the corona sheath is on the order of 10 μs , much faster than time discharge of the channel-base current, [13]. The outer radius of zone 1 primarily increase, reaching the maximum at $t \cong 1.5$ μs . Velocities of the outer surfaces of zones 1 and 2 are in the range of tens of kilometers per second, Fig.3. The rapid fall of $v^+_{out}$ starts at the very beginning of the return stroke. The maximum of this velocity is greater than the velocity of the zone 2 shrinkage ( $v^-_{out}$ ) but it rapidly decreases to zero value at approximately $t \cong 1.5$ μs . The shrinkage of zone 1 begins at that instant of time, accelerates for a while and then slowly diminishes to zero value.

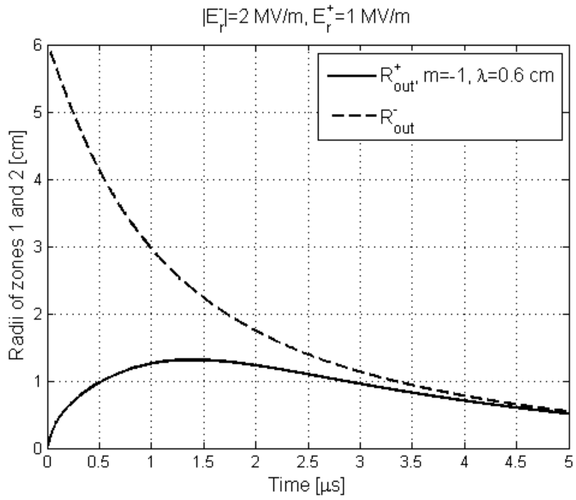


Figure 1. Radii of zones 1 and 2 vs time according to the GTCS model and (1) for stroke 1 in flash S0033, [13].

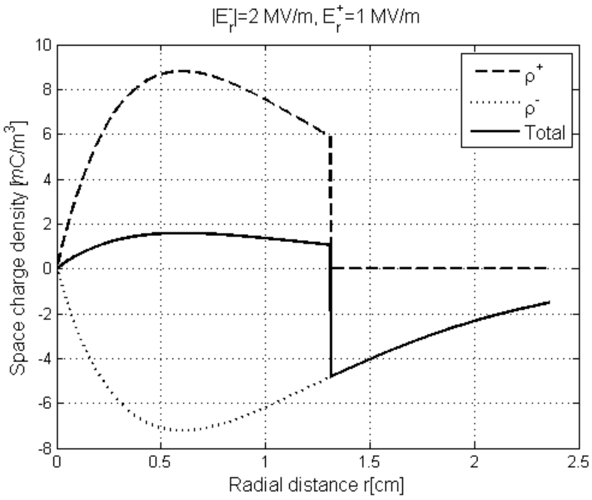


Figure 2. Space charge density in zones 1 and 2 vs radial distance in the corona sheath according to the GTCS model for stroke 1 in flash S0033, []. The graphs correspond to the instant of time $t \cong 1.5$ μs where the maximum of the radius of zone 1 occurs, Fig.1.

Fig.4 depicts the corona sheath electric field vs radial distance. The field increases approximately linearly in zone 1 from zero to 1 MV/m and then decreases to the value of -2 MV/m, as originally assumed in the corona sheath model. Second zero crossing of the field occurs inside zone 2. It stays inside that zone moving together with it during the discharge.

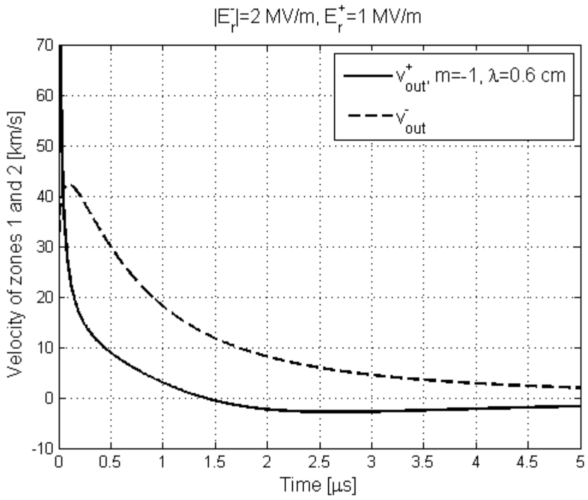


Figure 3. Velocities of the outer radii of zones 1 and 2 vs time according to the GTCS model for stroke 1 in flash S0033, [13].

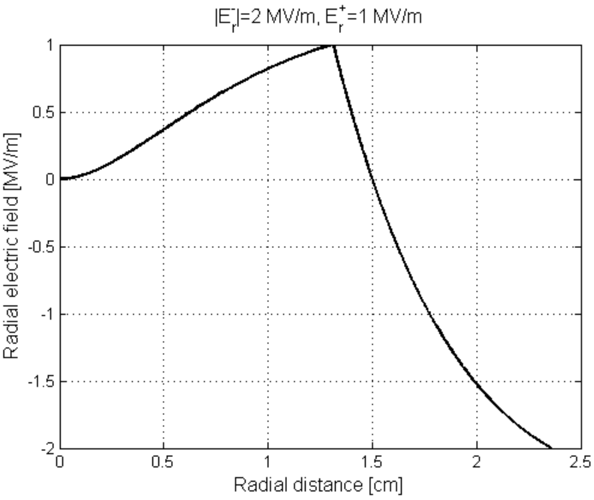


Figure 4. Radial electric field in zones 1 and 2 vs radial distance in the corona sheath according to the GTCS model for stroke 1 in flash S0033, [13]. The graph corresponds to the instant of time is $t \cong 1.5$ μs where the maximum of the radius of zone 1 occurs, Fig.1.

Figures 5-7 examine the influence of different parameter values in (1). In Fig. 5 the graphs of the initial (leader) negative space charge density distribution vs radial distance in the corona sheath are plotted for different $\lambda$ and $m$. It is possible to represent very different distributions including previously examined: the distribution with the constant space charge ( $\lambda \to \infty$ and $m = 0$ ) [6, 8], or the distribution with inverse proportionality to distance (i.e. with constant electric field, $\lambda \to \infty$ and $m = 1$) [10]. It should be noted that in both cases an unrealistic distribution near the channel core is assumed ( $\rho(r \to 0) \to 0$ is expected) unlike other distributions depicted in Fig. 5. Moreover, all distributions are normalized to the same return stroke using (2) and the data from table I.

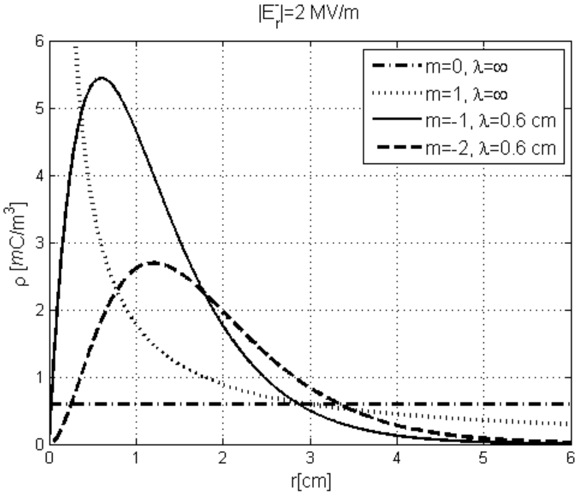


Figure 5. Initial (leader) negative space charge density distributions in the corona sheath according to the GTCS model for stroke 1 in flash S0033, [11].

Fig.6 depicts the time dependence of the zone 1 radii for different space charge distributions. The maximum and the minimum radii of $R_{out}^{+}$ correspond to the distributions with constant space charge and constant field inside the corona sheath, respectively.

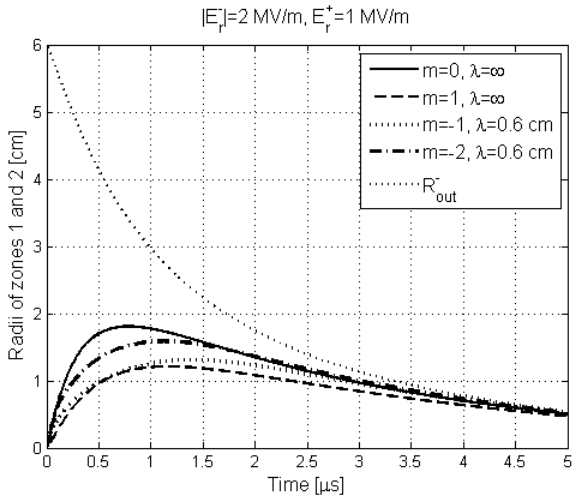


Figure 6. Time dependence of radii of zones 1 and 2 for different space charge distributions according to the GTCS model for stroke 1 in flash S0033, [11].

In Fig.7 the influence of the magnitude of the breakdown field at the outer boundary of zone 1 is examined. Small increase of the maximum of $R_{out}^{+}$ is observed with the great decrease of the magnitude of $E_r^{+}$ .

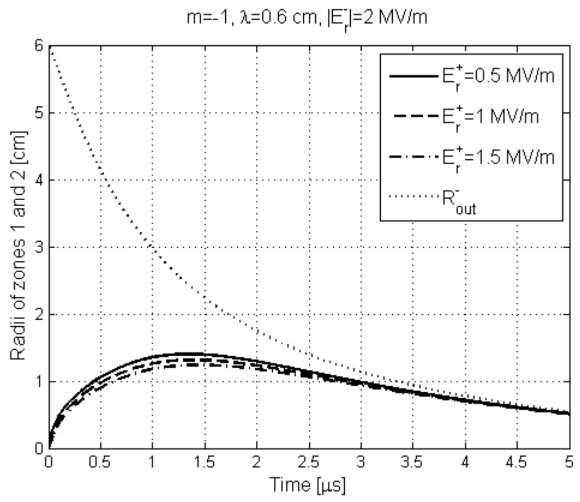


Figure 7. Time dependence of radii of zones 1 and 2 for different value of breakdown electric field at outer boundary of zone 1 according to the GTCS model for stroke 1 in flash S0033, [11].

## VI. CONCLUSION

We have numerically analyzed a new generalized equation for the space charge density distribution inside the channel corona sheath that represents a generalization of previously established space charge densities. Previously used corona charge densities are obtained as special cases of this distribution when particular values of the parameters are used. The influence of different distribution profiles on the corona sheath dynamics has been examined. Numerical simulations showed that the new generalized function possesses excellent features regarding flexibility in modeling different corona sheath charge density distributions. Furthermore, it is simple and easy for calculations. Results derived in this study can be used for further development of corona sheath models, as well as for the investigation of channel dynamics during the return stroke.

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