Lightning imaging with thunder using broadband
direction-of-arrival estimation technique

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Abstract—A single-station-based three-dimensional (3D) acoustic lighting mapping system comprising a microphone array has been developed and used for lightning observations, in which a new broadband direction-of-arrival (DOA) estimation techniques namely incoherent signal-subspace method are proposed for thunder signals in the far-field. Two cloud-to-ground (CG) flashes with highly branch channels recorded by the system are analyzed and presented in this paper. The acoustic mapping results can not only locate on the main channel produced by the return stroke process but also depict the branch channels. Moreover, the located thunder sources by the mapping system and the photography of the high speed camera agree well with each other demonstrating the effectiveness and correctness of the estimation techniques and the mapping system.

Keywords- thunder, lightning location, microphone array, broadband DOA estimation technique

I. INTRODUCTION

Lightning is an electromagnetic pulse phenomenon with high intensity causing all kinds of disasters frequently, which is concerned by many industries such as weather, electric power, aerospace, aviation, oil and so on because of its widespread influence [1-3]. The lightning location system based on the very low frequency (VLF) and high frequency (HF) electromagnetic signal has been widely used in the lightning engineering and scientific research for a long time [4]. It is well known that lightning can not only produce strong electromagnetic and optical signal, but also intense thunder signal, which can be analyzed for imaging the lightning discharge channel [5-6]. Using spatially separated receivers, lightning channel could be reconstructed from either the very high frequency (VHF) electromagnetic signal or the acoustic signal [7-8]. In the past decades, various VHF mapping systems, such as VHF interferometer and Lightning Mapping Array (LMA), have been developed, however, VHF technique has its inherent drawback which is not suitable to locate high current events (such as return strokes) because VHF radiation is produced by lightning breakdown processes. Audible thunder (about 20 Hz to 20 kHz) is generally associated with rapidly heated lightning channels, so thunder imaging could be used to locate high current events; moreover, it is suggested that not only return strokes(RS) but also other impulsive currents, such as in a leader step or a K-process could emit acoustic signals [9]. Therefore, thunder imaging may be another effective method to depict a more comprehensive lightning picture to complement VHF results.

Few firstly reconstructed the geometry of lightning channels from thunder data obtained with a Y-shaped array of microphones with the dimension of 30 meters in the 1970s. In the acoustic technique they divided a thunder record into a series of short contiguous time periods, and determined the direction of propagation of the predominant acoustic signal during each of the time periods by using cross-correlation analysis. Then kinds of arrays with different modeling and dimension were developed and the effects of wave front curvature, wind, and temperature on the determination of the direction of propagation of thunder was examined to increase the accuracy of lightning channel mapping [10-12]. In the last few years, with the development of techniques and hardware technology, the acoustic localization re-awoke much concern. Arecchi used acoustic (3.3–500 Hz) arrays to locate local (<20 km) thunder produced by triggered lightning and compared the acoustic source locations with those obtained by the Lightning Mapping Array (LMA) for estimating the accuracy [13]. Johnson used a network of broadband microphones, including a 4 - element array, to locate the sources of thunder occurring during an electrical storm and combined slowness search and distance ranging to identify thunder regions in three dimensions (out to 12 km) [14]. In China, Zhang developed a single-station-based lightning channel imaging system consisted of a small-size microphone array to observe the rocket-triggered lightning. The lightning channel imaging results had been compared with the photographs observed by a high-speed camera [15]. Qiu developed a lightning channel reconstruction system by combining a two-dimensional (2D) VHF broadband interferometer and a three-dimensional (3D) acoustic lightning mapping system [16]. Yang proposed an improved cross power spectrum phase method to calculate time delay by phase difference which did better in discerning the branch channel [17]. All of the above methods obtained the direction of propagation of the acoustic signal by the geometric relation of the array and the time delay of the thunder signal which was calculated by the cross correlation function method and the cross power spectrum phase method. Both of the two methods are the direction-of-arrival (DOA) estimation technique for the narrowband signal. However, the thunder signal belong to the broadband signal with different information for different frequency, therefore, the time delay cannot be expressed by the
time or the phase difference of the signal simply. Moreover, the resolution of the time delay estimation method is low and the estimation error will be increased under the low signal-to-noise ratio (SNR).

In this work, a new single-station-based lightning channel location system combined a cross microphone array is designed to observe the natural lightning, in which a new direction-of-arrival (DOA) estimation technique for broadband signals namely the maximum power beamforming algorithm is proposed. Two natural cloud-to-ground lightning are presented and the lightning location results are compared with the photographs by the high-speed camera. It is shown that this system can provide a practical three-dimensional observation imaging even for multi-branch lightning which has application value for studying lightning mechanism and monitoring small scale lightning.

II. INSTRUMENTATION AND METHODOLOGY

A. Instrumentation

The acoustic array used for thunder measurements consists of 19 electret microphones which are placed on a three-dimensional steel frame as shown in Fig. 1: twelve microphones are symmetrically distributed on the horizontal bar while seven microphones on the vertical bar. Based on the coordinate system established with the horizontal and vertical bars of the microphone array the coordinates of the 19 microphones are listed in Table 1. The microphones have a frequency response of 20 Hz – 100 kHz and a dynamic range of > 122 dB which are connected with shielded cable to the central logger NI PXIe-4492 and PXIe-4497, sampling at 50kHz. What’s more, FASTCAM-SA1.1 high speed video camera system developed by the Photron Company is set up nearby the acoustic array of which the fastest frame rate is 650 k frame/s and the maximum resolution is 1024×1024 pixels.

| X1=[0.25, 0, 0] | X2=[0.5, 0, 0] | X3=[1, 0, 0] |
| X4=[0.25, 0, 0] | X5=[0.5, 0, 0] | X6=[0, 0.25, 0] |
| X7=[0, -0.25, 0] | X8=[0, -0.5, 0] | X9=[0, -0.5, 0] |
| X10=[0, -1, 0] | X11=[0, 0.5, 0] | X12=[0, 1, 0] |
| X13=[-0.25, 0, 0] | X14=[-0.5, 0, 0] | X15=[-1, 0, 0] |
| X16=[-0.25, 0, 0] | X17=[-0.5, 0, 0] | X18=[-1, 0, 0] |
| X19=[-0.25, 0, 0] | X20=[-0.5, 0, 0] | X21=[-1, 0, 0] |

The trigger signal is generated by the optical signal detector when the voltage of the optical radiation signal exceeds the trigger threshold set in the internal trigger of the optical signal detector, which is then sent to the NI and high speed video to start the observation system as shown in Fig. 2, therefore, all records are synchronized in time.
B. Methodology

In order to reconstruct lightning channel from acoustic emissions, the direction-of-arrival (DOA) estimation technique is employed, which can be divided into the narrowband signal and the broadband signal DOA technique according to the different processing objects.

Assume that the center frequency, the highest and lowest frequency of the signal is \( f_c, f_H \) and \( f_L \) respectively, then the bandwidth is \( B = f_H - f_L \). It is generally believed that if the ratio of the bandwidth and the center frequency \( B/f_c \) is more than 0.1 the signal will be classified as the broadband signal, otherwise it will be considered as the narrowband signal. In general, the frequency of the thunder signal ranges from a few hundred Hertz to several thousand Hertz and the center frequency of the thunder signal varies from 210–280 Hz \([18-19]\), therefore, the thunder signal must be the broadband signal. In this paper, a new DOA estimation technique for broadband signals will be employed to obtain the direction of the thunder signal.

Since the dimension of the acoustic array (<1 m) is much smaller than the distances to the lightning channels (=kilometers), the thunder waves that generated by the lightning channel are propagated across the array can be accurately approximated by a set of plane waves. Therefore, the ray tracing method is divided into two steps: firstly, the phase velocity vectors for the set of plane waves can be determined by the broadband DOA estimation technique which will be described briefly in this section, and secondly we trace the vectors back to their origins through the time delay between the acoustic signals and the electromagnetic signals both of which can be detected by the microphone array.

In the first step, we divide the microphone array into two arrays: a planar array and a vertical line array. The azimuth of the vector is estimated from the thunder signals detected by twelve microphones on the planar array using DOA estimation technique while the elevation is obtained from the signals by seven microphones on the vertical array based on the same method namely the maximum power beamforming algorithm. Therefore, the two-dimensional estimation problem is transformed into one-dimensional problem, increasing the robustness of the algorithm and reducing the complexity of computation.

The maximum power beamforming algorithm is proposed by Tung [20] which is essentially an extended method based on the DOA estimation algorithm for the narrowband signal. The basic idea is to divide the broadband signal into a plurality of narrowband signals by Fourier transform, and then adopt the narrowband signal beamforming algorithm for each narrowband signal to generate DOA estimations. These estimations are averaged accordingly to obtain the DOA estimation of the broadband signal.

Assume that \( M \) broadband sources impinge on the array which is consisted of \( P \) sensors. The gain of each sensor is consistent in an ideal case and the signal received by the \( p \) sensor can be expressed as:

\[
x_p(t) = \sum_{m=1}^{M} S_m(t \cdot \tau_{mp}) + \omega_p(t) \quad p=1,2,\ldots,P
\]

where \( S_m(t \cdot \tau_{mp}) \) represents the \( m \)th acoustic signal arriving at the \( p \)th microphone, \( \omega_p(t) \) represents the noise signal of the \( p \)th microphone, in which \( S_m(t) \) and \( \omega_p(t) \) are the broadband signal and the broadband white noise respectively, \( \tau_{mp} \) represents the time in which the \( m \)th signal arrived at the \( p \)th microphone. Assume that \( M \) broadband signals have the same center frequency \( f_0 \) and bandwidth \( B = f_H - f_L \), in which \( f_H \) and \( f_L \) represent the highest and lowest frequency of signals respectively. Most of the algorithm processing the broadband array signal is to transfer the time-domain data into frequency domain data model by filter banks or discrete Fourier transform. Then (1) can be transformed into by Fourier transform with \( N \) sampling points:

\[
X_p(f_j) = \sum_{m=1}^{M} S_m(f_j)e^{j2\pi f_j \tau_{mp}} + W_p(f_j) \quad j=1,2,\ldots,J
\]

where \( J \) is the number of frequency band with non-overlapping and \( f_j \), \( f_H \) and \( f_L \), \( X_m(f_j) \) is the frequency-domain data received by the \( p \)th microphone, \( S_m(f_j) \) is the frequency-domain data of the \( m \)th acoustic signal, \( W_j(f_j) \) is the frequency-domain data of the noise received by the \( p \)th microphone. The data in frequency \( f_j \) of \( P \) microphones is arranged in a column vector: \( X(f_j) = [X_1(f_j), X_2(f_j), \ldots, X_P(f_j)]^T \), where \( [\cdot]^T \) is the symbol of transposition, then (2) can be expressed in matrix form:

\[
X(f_j) = A(f_j)S(f_j) + W(f_j)
\]

where the signal vector in frequency \( f_j \) \( S(f_j) = [S_1(f_j), S_2(f_j), \ldots, S_M(f_j)]^T \), the noise vector in frequency \( f_j \) \( W(f_j) = [W_1(f_j), W_2(f_j), \ldots, W_P(f_j)]^T \), \( A(f_j) \) is the orientation matrix in frequency \( f_j \) and can be expressed as:
Each column of the matrix is the orientation vector in frequency $f_j$, $a_{m}(f_j) = \begin{bmatrix} e^{i2\pi f_j \omega_m}, e^{i2\pi f_j \omega_m}, \ldots, e^{i2\pi f_j \omega_m} \end{bmatrix}^T, m=1,2,...M$. There is only one unknown the time delay $\tau$ which is obtained by the structure of the array and the angle of the sources. Fig. 3 shows the diagram of any two microphones in space in which the circular symbol “o” represents the microphone. Assuming that one microphone is located in the origin coordinate system and the coordinate value of the two microphone are $(0,0,0)$ and $(x,y,z)$ respectively. The time delay can be derived by geometric relations of the microphones, which can be expressed as:

$$\tau = \frac{(xcos\phi+y\sin\phi+z\sin\phi)}{c}$$  \hspace{1cm} (5)

where $c$ represents the speed of acoustic signal, $x, y$ and $z$ are x-value, y-value and z-value of one of the microphones, $\theta$ and $\varphi$ are the elevation and azimuth angle of the thunder sources respectively. Then put the expression of the time delay $\tau$ (5) into (3), the signal model can be rewritten as:

$$X(f_j) = A(f_j, \theta, \varphi)S(f_j) + W(f_j)$$  \hspace{1cm} (6)

where $\theta$ and $\varphi$ are the only two unknowns. $A(f_j, \theta, \varphi)$ will be abbreviated to $A(\Phi)$ in the following section. The covariance matrix of the array in frequency $f_j$ can be obtained according to (6):

$$R_{x}(f_j) = E[ X(f_j)X^H(f_j) ] = A(\Phi)R_{s}(f_j)A^H(\Phi) + R_{w}(f_j)$$  \hspace{1cm} (7)

where $E[\cdot]$ and $[\cdot]^H$ is the symbol of mathematical expectation and conjugate transpose. $R_{s}(f_j)$ is the covariance matrix of the signal in frequency $f_j$, $R_{w}(f_j)$ is the covariance matrix of the noise in frequency $f_j$. Then Under the hypothesis of Gauss white noise $R_{w}(f_j) = \sigma_{w}^2I$, (7) can be rewritten as:

$$R_{x}(f_j) = A(\Phi)R_{s}(f_j)A^H(\Phi) + \sigma_{w}^2I$$  \hspace{1cm} (8)

The signal will be divided into $D$ adjacent segments in the actual processing. The output vector in frequency domain of each segment is $X_d(f_j), d=1,2,...D$. Then the Covariance estimation $\hat{R}_{x}(f_j)$ in frequency $f_j$ can be obtained:

$$\hat{R}_{x}(f_j) = \frac{1}{D} \sum_{d=1}^{D} X_d(f_j)X_d^H(f_j)$$  \hspace{1cm} (9)

According to the covariance matrix estimation $\hat{R}_{x}(f_j)$ in the $j$th frequency band and the orientation matrix $a_{m}(f_j) = \begin{bmatrix} e^{i2\pi f_j \omega_m}, e^{i2\pi f_j \omega_m}, \ldots, e^{i2\pi f_j \omega_m} \end{bmatrix}^T$, the cost function is obtained:

$$\max_{(\theta,\varphi)} J(\theta,\varphi) = \sum_{k=1}^{N/2} A(f_j, \theta, \varphi)^H \hat{R}(k)A(f_j, \theta, \varphi)$$  \hspace{1cm} (10)

The direction of propagation of the acoustic signal will be obtained while the $J$ is the maximum and then the distance of the thunder sources can be calculated by the time difference between the acoustic signal and the electromagnetic signal. Therefore, the three-dimension location results of the thunder sources will be determined with the angle and the distance of the acoustic signals.

C. Observations and Results

During the summer from 2012 to 2015, an acoustic array and a high speed camera were deployed in the city of Wuhan, China to observe the natural lightning synchronously. The array were located at the top platform on a 7-floor building, with a relatively height of 24 meters or so, which was assigned as the origin point of the location sources. In order to verify the proposed method, the location results were compared with the photograph both of which were unified into the same coordinate system. Define the north of the observation as the x axis and the azimuth of the source in this direction was 0 degree, and the azimuth increased from 0 to 360 degree clockwise. Two natural CG lightning would be presented in this section. The deviation of the angle was about 1-2 degree caused by the distance between the acoustic array and the camera. In the next section, it will be shown that there is a parallel illumination channel beside the lightning channel in the lightning photographs which must be produced by the reflections from glass windows of the observation room.

1) CG 234291

The first CG 234291 occurred on June 15, 2014. The thunder signal measured at a single microphone and its spectrum distribution was shown in Fig. 3, from which we could see that there were two electromagnetic signals produced by the return stroke and the thunder lasted about 2 second. The lightning struck the tall building as shown in Fig. 4 with the distance of 660m which was similar with the distance of 680m calculated from the time delay between the electromagnetic signal and the thunder signal. The lightning had only one channel without any complex branch channel as seen in the photograph, and the azimuth of the lightning location point was about 276 degree. The angle results of the acoustic sources obtained by the new technique and the photograph were shown in the same coordinate system as shown in Fig. 5 in which the red point and the blue point represented the acoustic sources and the optical sources respectively. It was shown that they were in good agreement with each other at the upper and

![Figure 3. The diagram of any two microphones in space](image-url)
bottom part of the lightning channel except there were a few error points near the tortuous channel from the comparison results which proved the feasibility and validity of the method. Meanwhile, the three-dimension locations could depict the lightning discharge channel clearly as shown in Fig. 6.

Figure 4. The thunder signal measured at a single microphone and its spectrum distribution of CG 234291

Figure 5. The photograph of the CG 234291 by the high-speed camera

2) **CG 135747**

The second CG 135747 also occurred on June 15, 2014. Fig. 7 showed the signal measured at a single microphone in which we could see that the time difference of arrival between the electromagnetic signal and the thunder signal was about 2 second and the thunder signal lasted also about 2 second. The lightning also struck the tall building facing the observation point as shown in Fig. 9 which was the photograph of the return stroke of the lightning. The difference between the two CG lightning was that this lightning was multi-branch lightning as shown in Fig. 8 which was filmed before the return stroke. It was shown that the lightning developed along three branch channels and connected the ground in the right side of the three channels. The three channel branching structure increased the difficulty of the location system because the thunder signal from different channels might interfere with each other. The acoustic location results were compared with the first photograph as shown in Fig. 10 in which the acoustic sources portrayed two branches which were in good agreement with the optical sources. The left two channels were downward leaders one of which was depicted by the location system. However,
there was no thunder source locations in the middle branch channel because the thunder signals of this channel might be submerged in the signals of the return stroke which produced stronger signal. It was proved that not only the return stroke but also the initial process could produce the thunder signals which could be detected by the microphone array. Fig. 11 showed the comparison between the acoustic location results and the second photograph in which the located results could cover the whole return stroke channel which also verified the effectiveness of the thunder imaging technique even for multi-branch lightning. What is more, we obtained the three-dimension locations as shown in Fig. 12 in which there were also two branch channels.

Figure 8. The thunder signal measured at a single microphone and its spectrum distribution of CG 135747

Figure 9. The first photograph of the CG 135747 by the high-speed camera

Figure 10. The second photograph of the CG 135747 by the high-speed camera

Figure 11. The comparison between the acoustic results and the first photograph of the CG 135747

Figure 12. The comparison between the acoustic results and the second photograph of the CG 135747
In this paper, a direction-of-arrival (DOA) estimation technique namely incoherent signal-subspace method (ISM) are firstly applied for locating the thunder sources and we set up a single-station-based three-dimensional (3D) acoustic lightning mapping system comprising a microphone array and a high-speed camera to observe the nature lightning.

In order to verify the proposed method, the location results were compared with the photograph both of which were unified into the same coordinate system. This work had shown coincident localization of acoustic and photograph sources from natural lightning for two events. The comparison results indicated that acoustic techniques were useful and effective because the localization of lightning events by the acoustic emissions and the high-speed camera agree well with each other for the two lightning. It was shown that this method could not only distinguish the different channel simultaneously in the same process and also image the high-current pulses during the downward leader process.

The basic criticism of the thunder ranging technique is that there were several electromagnetic signals produced by the different return strokes, however, the thunder signals almost arrived at the microphone array at the same time which caused a problem that it was difficult to match the electromagnetic signal and the thunder signal correctly. Meanwhile, to describe the acoustic propagation properly and to locate the lightning channel precisely, we should obtain the temperature and wind profile throughout the region of the experiment at the time of lightning event. Such detailed information is not readily obtained.

REFERENCES