Comparison of two techniques of calculating electromagnetic fields from lightning

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***Abstract*— A comparison is made between the electric fields calculated using dipole approximation and the ones calculated using the charge acceleration equations. Even though the same final result is obtained for the electric fields, the various field components, such as radiation, induction and static, that add up to the total electric field were different in the two formulations. It is shown that the signature of these field components depend not only on the basic technique used in estimating the electric fields but also on the way in which the problem is formulated.**

Keywords- Lightning, Electromagnetic fields, radiation fields, accelerating charges.

I. Introduction

In the literature, three procedures have been used to calculate the electromagnetic fields from return strokes [1, 2]. In the first technique of these, the source is described only in terms of current density and the fields are expressed entirely in terms of the return stroke current. In the second technique, the source is expressed in terms of the current and the charge densities and the fields are given in terms of both the current and the charge density. In the third technique, the fields are expressed in terms of the apparent charge density. The fields are connected to the source terms through the vector and scalar potentials. A fourth technique was introduced recently by Cooray and Cooray [3]. In this technique the standard equations for the electromagnetic fields generated by accelerating charges are utilized to evaluate the electromagnetic fields from lightning return strokes. Cooray and Cooray [3] showed how this technique can be applied to calculate the electromagnetic fields predicted by different return stroke models. Cooray and Cooray [4] also applied these equations to re-derive the electromagnetic fields from short dipoles. This technique has several advantages over other conventional methods of electromagnetic field calculations when the system under consideration can be represented by a set of transmission lines in air. In this case the complete electromagnetic field can be reduced to a sum of electromagnetic fields generated by the changes in currents taking place at the end points of the transmission lines. For this reason this technique will reduce the computational time necessary for field calculations and at the same time gives a clear physical picture of the events that led to the various parts of the electromagnetic fields. In this paper we will compare the dipole approximation against the fields calculated using the charge acceleration equations.

II. Theory

In the analysis we will consider the transmission line model of return strokes [5]. In the transmission line model a return stroke is represented by a current pulse that propagates from ground to cloud with uniform speed and without attenuation. In order to simplify the analysis it is assumed that the return stroke speed is equal to the speed of light. Moreover the lightning channel is assumed to be straight and vertical. We also assume that once the current is initiated at the ground end it will continue to propagate upwards for a time which is longer than the duration of the fields of interest. Otherwise, the equations to be presented later have to be modified to take into account the termination of the current at the cloud end of the channel.

In this paper we will utilize three procedures to evaluate the electric fields produced by the transmission line model and a comparison is made between the results obtained using these different procedures. In the first procedure, the equations pertinent to charge acceleration equations are applied to the whole channel treating it as a one single source of electromagnetic fields. In the second procedure the channel is divided into large number of elementary channel sections and the electric field is calculated by treating each of these channel elements as a short dipole. The standard dipole field expressions are used to write down the electric field. In the third procedure the channel is divided into elementary channel sections as before but the electric field produced by each channel element is obtained by treating the channel element as an electric field source in which a current pulse is initiated at one end, travels along the channel element with the specified speed (speed of light in our case) and is terminated at the other end. The field produced by the channel element is obtained using charge acceleration equations and the total field is obtained by summing the contributions from each channel element.

In the present paper the electric fields are calculated at ground level at a horizontal distance d from the lightning channel. The relevant geometry is shown in Figure 1.

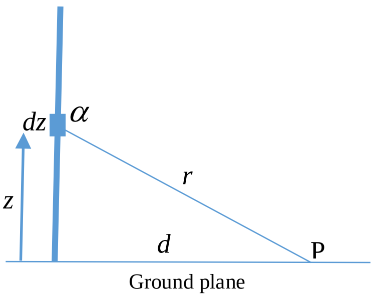

Figure 1: Geometry relevant to the calculation of electric field generated by a lightning channel located over a perfectly conducting ground plane. The channel is located along the z-axis.

In the case of charge acceleration equations the field components can be separated into radiation, velocity and static terms. The radiation terms are arising due to the acceleration of charges, velocity term is the Lorentz transformed Coulomb field produced by the charges propagating with uniform speed and the static field is the Coulomb field produced by stationary but time varying charges located on the channel. In the case of current pulses propagating with the speed of light the velocity term becomes zero [3]. In the case of dipole technique the field components can be separated into radiation term, induction term and static term. In the next section we will give the mathematical expressions for the electric field components as dictated by different techniques.

A. Technique 1: Charge acceleration equations applied to the whole channel

Here the total electric field becomes pure radiation. This is the case because the velocity term is zero due to the fact that the pulse propagation speed is equal to the speed of light and the static term is zero because there is no charge accumulation at the ground end of the channel due to the fact that it is connect to a perfectly conducting ground (i.e. the real charge and the image charge cancel each other at the grounding point). This total electric field, which is pure radiation, at ground level is given by

1) Radiation

$$E_{z,total}(t) = -\frac{i(t-d/c)}{2\pi\varepsilon_o c r_2} \qquad (1)$$

In the above equation $i(t)$ is the current injected into the channel at ground level.

B. Technique 2: The dipole technique

In this technique the channel is divided into a large number of elementary sections and each channel element is treated as a short dipole. The dipole gives rise to three field components, namely, the static, induction and radiation. The total field is the sum of fields due to all the dipoles. The total electric field at a distance d from the point of strike is given by (H is the height of the channel at the time of interest)

1) Static field:

$$E_s(t) = \frac{1}{2\pi\varepsilon_0}\int_0^H \frac{2-3\sin^2\alpha}{r^3}\int_{t_b}^{t} i(z,\tau - r/c)\,d\tau\, dz \qquad (2)$$

2) Induction field

$$E_i(t) = \frac{1}{2\pi\varepsilon_0}\int_0^H \frac{2-3\sin^2\alpha}{cr^2} i(z,\tau - r/c)\,dz \qquad (3)$$

3) Radiation field

$$E_s(t) = \frac{1}{2\pi\varepsilon_0}\int_0^H \frac{\sin^2\alpha}{c^2 r}\frac{\partial i(z,\tau - r/c)}{\partial t}\,dz \qquad (4)$$

The total electric field at the point of observation is given by

$$E_{z,total}(t) = E_r(t) + E_i(t) + E_s(t) \qquad (5)$$

In the above equations $i(t,z)$ is the current at height z.

It is of interest to point out that Thottappillil et al. [6, 7] has shown that, for the special case of transmission line model with speed of light, this sum will reduce to a pure radiation term given by (1) as obtained using the charge acceleration equations.

C. Technique 3: Charge acceleration equations applied to each channel element

In this procedure the channel is divided into a large number of small elements and the total field is constructed by summing up the contributions from all the channel elements. The field created by each channel element consists of two components, a radiation field and a static field. The radiation field is generated by the acceleration and deceleration of charges at the two ends of the channel element and the static field is created by the accumulation of opposite charges at the two ends of the channel element. Since the speed of propagation of the current is equal to the speed of light no velocity term is present in the calculations.

The electric field at a distance d due to the sum of the electric fields created by the elementary channel sections can be written as

1) Field components resulting from the static fields produced by charges at the end of the elementary channel sections

$$E_s(t) = -\frac{1}{2\pi\varepsilon_0}\int_0^H \frac{\left(1-3\cos^2\alpha\right)}{r^3}\int_{r/c}^{t} i(z,\tau - r/c)\,d\tau dz - \frac{1}{2\pi\varepsilon_0}\int_0^H \frac{\cos\alpha(1-\cos\alpha)}{cr^2} i(z,\tau - r/c)dz \quad (6)$$

2) Field components resulting from the radiation fields generated by the charge acceleration and deceleration at the ends of the elementary channel sections

$$E_r(t) = -\frac{1}{2\pi\varepsilon_0}\int_0^H \frac{(1+\cos\alpha)(1-2\cos\alpha)}{cr^2} i(z,\tau - r/c)dz + \frac{1}{2\pi\varepsilon_0}\int_0^H \frac{\sin^2\alpha}{c^2 r}\frac{\partial i(z,\tau - r/c)}{\partial t}dz \quad (7)$$

The total electric field at the point of observation is given by

$$E_{z,total}(t) = E_r(t) + E_s(t) \quad (8)$$

III. RESULTS AND CONCLUSIONS

First, observe that the physical nature of the field components (i.e. their separation into static, induction, radiation etc.) is different in the different techniques. Interestingly, even when one consider the charge acceleration equations the division of the fields into different components differ depending on whether the whole channel is treated as one radiating unit or whether the channel is divided into a large number of elements and the fields due to each element is obtained using charge acceleration equations. Second, observe that in the latter technique the electric field arising from the radiation emanating from the channel ends contains not only a $1/r$ term (which is proportional to the current derivative) but also a term that varies as $1/r^2$ (which is proportional to the current). The origin of these two terms is the following. The net radiation field produced by a channel element is the sum of radiation fields produced by charge acceleration at one end due to current initiation and charge deceleration at the other end due to current termination. These terms, having opposite signs, are proportional to the current and vary as $1/r$. However, the sum of these will generate two terms one varying as $1/r$ which is proportional to the current derivative and the other varying as $1/r^2$ which is proportional to the current. So the physical process that gives rise to $1/r$ and $1/r^2$ terms is the same. Note that the individual radiation fields generated by charge acceleration and deceleration are proportional to the current and the term proportional to the current derivative arises when one takes the sum of the fields. Thus, both acceleration and subsequent deceleration are necessary to generate the radiation field proportional to the current derivative term. When only one term is present, as in the case of transmission line model without current termination, the radiation field will be proportional to the current itself. A similar situation exists in the static field. The static field is the sum of two terms one produced by charges accumulated at the end of the channel when the current is initiated and the other produced by opposite charge that is accumulated at the other end of the channel when the current is terminated. Both these fields, having opposite sign, vary as $1/r^2$ since they are nothing but Coulomb fields. However, the total static field is the sum of these terms and this sum will give rise to a field component that varies as $1/r^3$. So the physical process that gives rise to $1/r^2$ and $1/r^3$ terms is the same. Third, observe that if one combines the $1/r^2$ term of the static field and the $1/r^2$ term of the radiation field the result is the same as the induction field of the dipole technique. Fourth, observe that the expressions for the electric fields derived using the charge acceleration equation differ depending on whether the whole channel is treated as one radiating unit or whether the channel is divided into a large number of elements and the total field is constructed by adding the contributions from all the elements. This shows that it is not only the technique utilized in field calculation but also the way in which the problem is formulated can change the nature of the various expressions that appear in the total field.

Even though the mathematical expression for the electric field given in equation 1 is different to the field components given by equations 2, 3 ,4 and 6, 7, all these expressions give rise to the same total electric field. For example the electric field at a distance of 100 m as predicted by equation 1 is shown in Figure 2. The electric field predicted by equation 5 and the three components of this field given by equations 2, 3 and 4 are shown in Figure 3. Note that both 1 and 5 predict the same total electric field. Another interesting point is that in 1 the total field appears as a radiation term whereas in 5 it is created by the sum of static, induction and radiation components.

In the analysis presented here it was assumed that the speed of propagation of the current pulse is equal to the speed of light. This made the analysis much simpler because the velocity field term is zero under this conditions. Of course, one can analyze the problem using a current speed that is less than the speed of light. Even in this case the two techniques predict the same total electric and magnetic fields even though the signatures of the field components that add up to the total field are different.

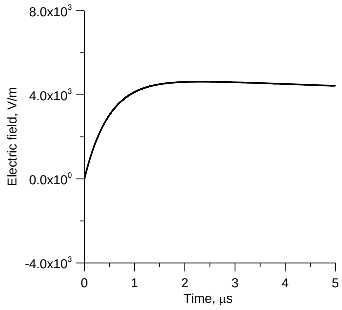

Figure 2: The electric field at ground level at a distance of 100 m from the strike as predicted by equation 1. The current at the channel base is assumed to be given by $I(t) = I_0 \left\{ e^{-t/\tau_1} - e^{-t/\tau_2} \right\}$ with I_0=8.18 kA, τ_1=50 μs, τ_2 = 0.5 μs.

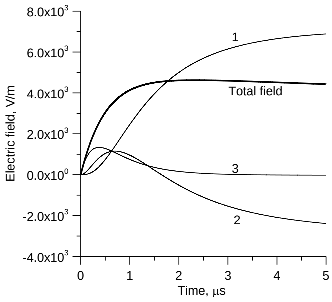

Figure 3: The total electric field at ground level at a distance of 100 m from the strike as predicted by equation 5. The three components, namely, static, induction and radiation are also shown separately by curves 1, 2 and 3 respectively. The current at the channel base is assumed to be given by $I(t) = I_0 \left\{ e^{-t/\tau_1} - e^{-t/\tau_2} \right\}$ with I_0=8.18 kA, τ_1=50 μs, τ_2 = 0.5 μs.

It is important to mention that the first three techniques mentioned (dipole, monopole and apparent charge density) are general methods to calculate remote electromagnetic fields from any charge/current distribution in space and time. The method pertinent to charge acceleration equations as presented in this paper is applicable to travelling charge/current distributions with certain speed. Work is underway to generalize this technique to calculate fields from arbitrary charge/current distributions.

CONCLUSION

The results presented in this paper show that it is not only the electric field calculating technique (i.e. dipole technique of charge acceleration equations) that could influence the signature of various field components that constitute the total electric field but also the way in which the problem is formulated.

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