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Grounding Modeling Using Transmission Line Theory: Extension to Arrangements Composed of Multiple Electrodes

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***Abstract*—This paper presents the application of transmission line theory to analyze grounding arrangements composed of multiple parallel electrodes. The grounding system is treated as a multiconductor transmission line and a solution method based on a similarity transformation is used to solve the resultant equations. The impulse response of grounding arrangements composed of multiple grounding rods and horizontal electrodes are simulated, and the results are presented in terms of the grounding potential rise and impulse coefficient. The accuracy of the results is assessed by comparison with simulations of the same cases using an electromagnetic field approach. The proposed model is useful to estimate quantities related to the lightning response of typical grounding arrangements of transmission and distribution lines.**

Keywords-grounding modeling; multiconductor transmission line, lightning response of grounding; transmission and distribution lines.

I. INTRODUCTION

Grounding has an important role in the lightning performance of transmission and distribution lines. Typically, grounding arrangements of high-voltage transmission lines (TLs) are composed of a set of horizontal electrodes, called counterpoise cables, whose length is selected depending on the value of soil resistivity [1]. In TLs, lower values of grounding impedance are aimed in order to ensure that line outages due to backflashover mechanism be within permissible levels. In case of overhead distribution systems, typical grounding arrangements consist of very simple configurations of concentrated electrodes, most commonly comprising one short vertical electrode, called ground rod, or a few aligned rods [1]. Such grounding arrangements are commonly installed along medium-voltage lines, in addition to poles supporting equipment such as transformers, and are usually connected to surge arresters responsible for protecting the equipment [1].

In a simplified approach, horizontal and vertical grounding electrodes are frequently treated in literature as a transmission line buried in soil, and the harmonic grounding impedance is computed as its input impedance [2], 0. However, in most works, transmission line theory is used to assess the lightning performance of only single horizontal or vertical electrodes [4], [5]. In case of more complex grounding arrangements composed of a set horizontal or vertical electrodes, a lumped-circuit approximation is often used and the TL equations are not solved directly [6].

The aim of this work is to extend transmission line theory to assess grounding systems composed of multiple parallel electrodes, using a solution method based on a similarity transformation. The proposed approach can be useful to deal with typical grounding configurations of transmission and distribution lines, providing accurate results within certain limits.

II. GROUNDING MODELING USING TL THEORY

A. Single Electrodes

The equations for a TL of a single conductor positioned along the *z* axis can be written in frequency domain as [7]:

$$\frac{d^2}{dz^2}V(z) = ZYV(z) \tag{1a}$$

$$\frac{d^2}{dz^2}I(z) = YZI(z) \tag{1b}$$

In (1), $V(z)$ and $I(z)$ are the phasors of voltage and current along the line, respectively. The per-unit-length impedance, *Z*, and admittance, *Y*, are

$$Z = j\omega L \tag{2a}$$

$$Y = G + j\omega C \tag{2b}$$

where *L*, *G* and *C* are the per-unit-length inductance, conductance and capacitance, respectively (the longitudinal

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resistance per unit length R is neglected, since its value is not important for the analysis of grounding electrodes). Considering the classical expressions proposed by Sunde, the per-unit-length conductance of a horizontal wire buried in soil can be computed by [8]

$$G = \frac{\pi}{\rho}\left[\log\frac{2\ell}{\sqrt{2da}} - 1\right]^{-1} \tag{3}$$

where ρ (in Ωm) is the soil resistivity, ℓ (in m) is the electrode length, a (in m) is the electrode radius, and d (in m) is the burial depth. The grounding capacitance is computed by considering the duality relationship between C and G

$$\frac{C}{G} = \rho\varepsilon \tag{4}$$

where ε is the soil permittivity (in F/m). Finally, the inductance is computed by

$$L = \frac{\mu}{2\pi}\left[\log\frac{2\ell}{a} - 1\right] \tag{5}$$

where μ is the soil permeability (in H/m).

In the case of buried vertical wires (rods), the grounding conductance is computed by

$$G = \frac{2\pi}{\rho}\left[\log\frac{4\ell}{a} - 1\right]^{-1} \tag{6}$$

being the capacitance computed using the same dual relationship given by (4), and the inductance given by (5).

Once computed the per-unit-length parameters, the TL equations are solved by incorporating the two following terminal conditions: 1) One end of the line is fed by a time-harmonic current of 1 A in the frequency range of interest for the transient study; 2) The other end of the line is open. Then, the harmonic grounding impedance $Z_T(j\omega)$ is computed as the input impedance of the line at each frequency, which is given by

$$Z_T(j\omega) = \frac{V(0)}{1A} \tag{7}$$

where $V(0)$ is the phasor of voltage at the sending end of the line, which is fed by the harmonic current. Finally, the Grounding Potential Rise (GPR) at feed point $v(t)$ in response to a impressed current $i(t)$ can be computed by

$$v(t) = \mathscr{F}^{-1}\left\{Z_T(j\omega)\mathscr{F}\left[i(t)\right]\right\} \tag{8}$$

where $\mathscr{F}$ and $\mathscr{F}^{1}$ denotes Fourier and inverse Fourier transform, respectively.

B. Multiple Electrodes

The equations for a TL of n parallel conductors positioned along z axis can be written in frequency domain as [7]

$$\frac{d^2}{dz^2}\hat{V}(z) = \hat{Z}\hat{Y}\hat{V}(z) \tag{9a}$$

$$\frac{d^2}{dz^2}\hat{I}(z) = \hat{Y}\hat{Z}\hat{I}(z) \tag{9b}$$

In (9), $\hat{V}(z)$ and $\hat{I}(z)$ are $n \times 1$ column vectors containing the n line voltages and currents. The $n \times n$ per-unit-length impedance, $\hat{Z}$, and admittance, $\hat{Y}$, matrices are given by

$$\hat{Z} = j\omega\hat{L} \tag{10a}$$

$$\hat{Y} = \hat{G} + j\omega\hat{C} \tag{10b}$$

where $\hat{L}$, $\hat{G}$ and $\hat{C}$ are the inductance, conductance and capacitance matrices, respectively. The diagonal elements of such matrices are computed using the expressions presented in the previous sub-section. The off-diagonal elements, representing the mutual effects, are computed using the same expressions, but replacing the radius by the horizontal distance between the buried conductors [8], [9].

Note that equations (9) are coupled because ZY and YZ are full matrices. It means that voltages and currents of one line conductor affect voltages and currents of all remaining line conductors. A classical solution to decouple such equations is to use a similarity transformation [7]. This transformation defines a change of variables between the actual voltage and current phasors and the so called modal voltages and currents. By applying this change of variables, equations (9) are decoupled into the form of n separate equations describing n isolated single-wire TLs [7]. Then, the techniques mentioned in previous sub-section is applied to each one of these individual single-wire TLs. Finally, another change of variables is used to return to the original voltage and current phasors, and time-domain results are obtained by using the Fourier transform.

III. DEVELOPMENTS

The developments of this work consisted in simulating the impulse response of vertical and horizontal grounding arrangements in terms of the grounding potential rise (GPR) associated with the injection of an impulsive current. In all analysis, a Heidler current pulse with a normalized peak amplitude of 1 kA and a virtual front time t_{d30} = 0.625 μs (measured as the time from $0.3I_p$ to $0.9I_p$, where I_p is the current peak value) and time to hal-value of 30 µs was assumed. This current waveform can be considered representative of median subsequent stroke currents measured at short instrumented towers [10]-[12].

Three values of low-frequency soil resistivity $\rho_0=1/\sigma_0$ were considered: 300, 1000, 3000 Ωm. The frequency dependence of soil parameters was determined using the Alipio-Visacro causal model proposed in [13] and expressed by (11) and (12).

$$\sigma = \sigma_0 + \sigma_0 \times h(\sigma_0)\left(\frac{f}{1\ \text{MHz}}\right)^{\gamma} \tag{11}$$

$$\varepsilon_r = \frac{\varepsilon'_\infty}{\varepsilon_0} + \frac{\tan(\pi\gamma/2)\times 10^{-3}}{2\pi\varepsilon_0 (1\text{MHz})^{\gamma}}\sigma_0 \times h(\sigma_0)\, f^{\gamma-1} \tag{12}$$

In (11) and (12), σ is the soil conductivity in mS/m, σ_0 is the low-frequency conductivity (100 Hz) in mS/m, ε_r is the relative permittivity, $\varepsilon'_\infty/\varepsilon_0$ is the relative permittivity at higher frequencies, ε_0 is the vacuum permittivity ($\varepsilon_0 \cong 8.854\times10^{-12}$ F/m) and f is the frequency in Hz. In this work, the following parameters were adopted in (11) and (12) to obtain mean results for the frequency variation of σ and ε_r, as recommended in [13]: $\gamma = 0.54$, $\varepsilon'_\infty/\varepsilon_0 = 12$ and $h(\sigma_0) = 1.26\times\sigma_0^{-0.73}$.

In order to assess the accuracy of the results provided by the proposed transmission line theory, the same grounding arrangements were simulated using the Hybrid Electromagnetic Model (HEM), which is based on electromagnetic field theory [14].

IV. Results

A. Vertical Grounding Arrangements

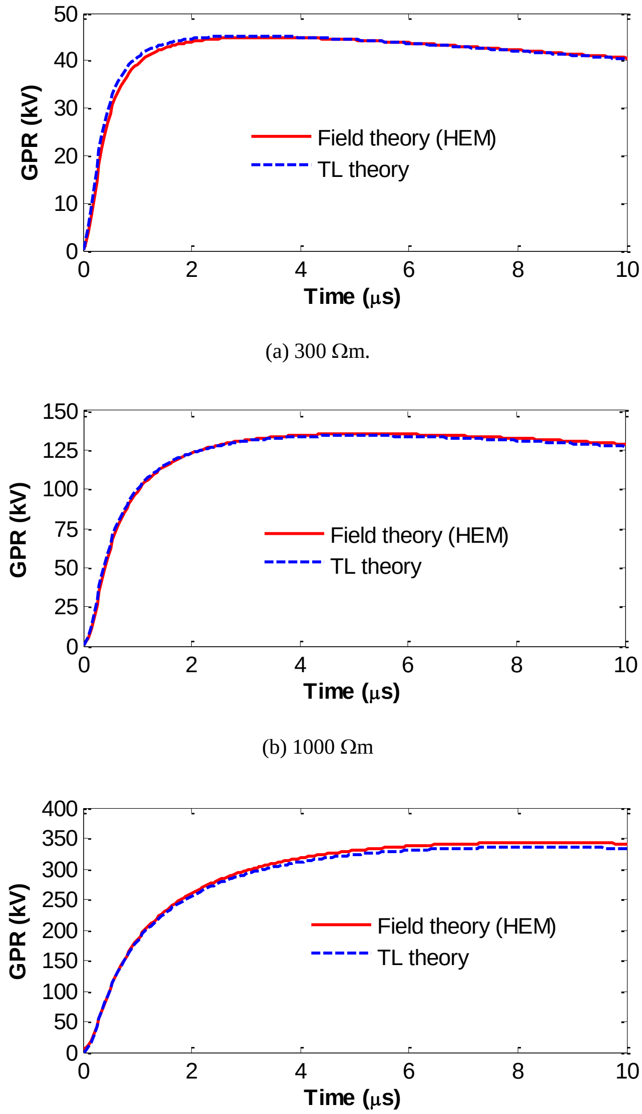
Vertical grounding arrangements are usually composed of concentrated electrodes, comprising short aligned grounding rods. Here, a configuration composed of three vertical grounding rods of 3 m forming an equilateral triangle with sides of 1 m is considered. The injected current is assumed to divide equally between the three rods.

Fig. 1 illustrates the GPR obtained by using the HEM model and the transmission line theory, considering the three different values of ρ_0. As can be observed, in all cases the results obtained by TL theory are in very good agreement with those computed by field theory (HEM model). Note that this result was somehow expected due to the short length of the electrodes, as discussed in [2] for a single rod. On the other hand, it should be stressed that the arrangement here analyzed is composed of three rods, and the conductive, capacitive and inductive couplings among them were properly taken into account by TL theory.

B. Horizontal Grounding Arrangements

In the case of horizontal grounding arrangements, it is common to use longer electrodes, depending on the value of soil resistivity. Here, a configuration composed of two parallel electrodes of length ℓ, both buried 0.5 m deep in soil, and set 20 m apart is considered. The impressed current is assumed to divide equally between the two electrodes.

Fig. 2 illustrates the GPR obtained considering the field and transmission line theories, for a soil resistivity of 3000 Ωm and 10-m electrodes. Again, as can be noted, the results obtained by TL theory are in very good agreement with those computed by field theory (HEM model), showing a difference of around 5%. Very similar results are obtained considering other values of low-frequency soil resistivity.



(a) 300 Ωm.

(b) 1000 Ωm

(c) 3000 Ωm

Figure 1. Grounding Potential Rise of an arrangement composed of three vertical grounding rods of 3 m forming an equilateral triangle (side = 1 m), subjected to an impulsive current.

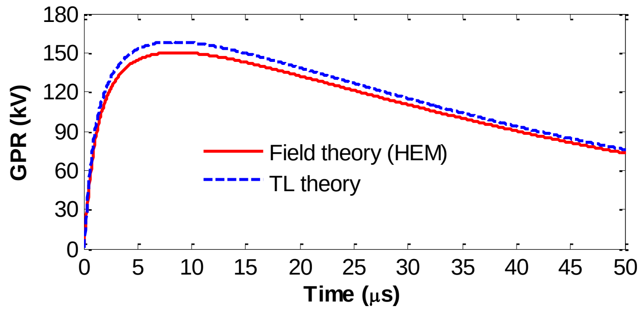


Figure 2. Grounding Potential Rise of an arrangement composed of two parallel electrodes of length (10 m each), both buried 0.5 m deep in a soil of 3000 Ωm, and set 20 m apart, subjected to an impulsive current.

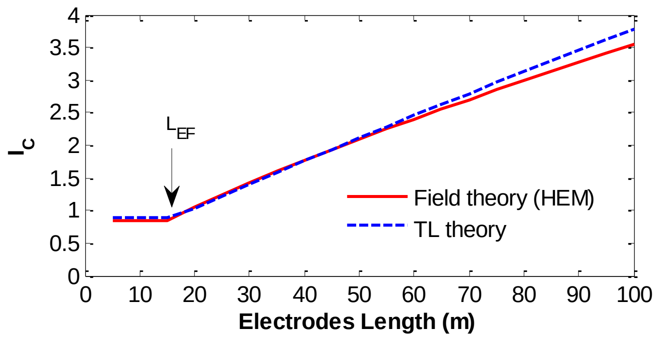
In order to make the analysis more comprehensive, the length of the electrodes is varied from 5 m to 100 m. For each arrangement, the GPR is simulated and then the impulse impedance Z_P, given by the ratio of the peak values of GPR and impressed current, is computed. Finally, the impulse coefficient I_C, given by the ratio of Z_P and the low-frequency resistance R_{LF}, is determined. The final results are presented in terms of curves of I_C *versus* ℓ. Note that such curves summarize the impulse grounding behavior. Furthermore, they are very important in industry applications since, in most cases, measuring the impulse impedance is not feasible in field conditions and the common practice consists simply in measuring the low-frequency resistance. Thus, the knowledge of the ratio $I_C = Z_P/R_{LF}$ allows developing estimates of the lightning response of the grounding systems from the measured R_{LF} [15].

Fig. 3 depicts the graphs of I_C *versus* ℓ. As a general behavior, the curves of I_C show initially a constant value while the electrode length is increased until it reaches the effective length (L_{EF}), defined by the point of inflection in the curves. After L_{EF} is reached, the impulse coefficient rises continuously with increasing the electrode length, since Z_P remains constant while R_{LF} decreases continuously. A detailed and comprehensive analysis of the impulse behavior of grounding electrodes can be found in [16].

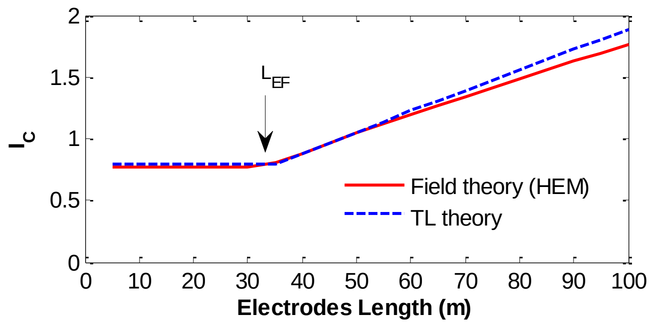
According to Fig. 3, the results obtained by using TL theory are in good agreement with those obtained by using the field approach. In case of longer electrodes, the values of I_C estimated by TL theory are slightly higher than those estimated by HEM. This is attributed to the overestimation of the inductive effect when using TL theory, which leads to an increase of the computed impulse impedance. It should be also stressed that the estimated effective lengths given by TL theory are in very good agreement with the results given by HEM. This is a relevant finding since the effective length is of paramount importance to the grounding design for lightning protection.

V. CONCLUSIONS

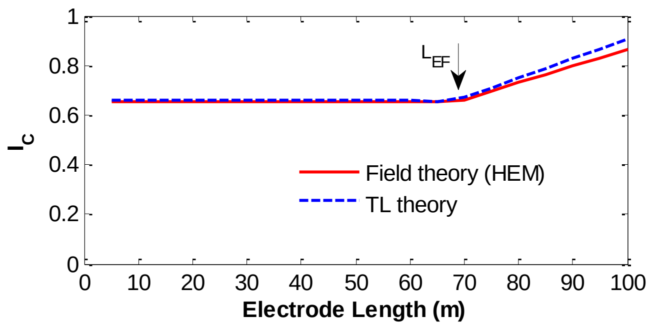
This work presents results of application of the TL theory to analyze grounding arrangements composed of multiple horizontal and vertical electrodes, using the classical approach of applying a similarity transformation to the transmission line equations. The frequency dependence of soil parameters was included in all analyses. The accuracy of the results obtained using the proposed TL approach was verified by means of comparisons with simulations of the same cases with an electromagnetic field approach. The proposed methodology is useful to evaluate the performance of actual grounding configurations of transmission and distribution lines. In particular, it is useful in industry applications since TL theory can be easily implemented with a low computational cost. Also, the proposed grounding model can be included in time-domain transients programs by means of an equivalent circuit or by using the modal-domain transmission line model of Marti extended to include frequency-dependent parameters as outlined in [17].



(a) 300 Ωm.



(b) 1000 Ωm



(c) 3000 Ωm

Figure 3. Impulse coefficient $I_C=Z_P/R_{LF}$ of an arrangement composed of two horizontal electrodes buried 0.5 m and set 20 m apart, subjected to an impulsive current.

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