



Correlation between Leader Charge and Peak Current as Depicted by the Self Consistent Return Stroke Model

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Abstract—The self consistent return stroke model proposed in an earlier work [6], had successfully predicted the basic features of current evolution in the channel, as well as, that in the far and near electromagnetic fields. A further validation of this model is taken up in this paper by deducing the relationship between the leader charge and the peak return stroke current. In this model the cloud is modeled as ellipsoidal structures and the leader is modeled as a thin wire. The charge distribution on the leader and its radius is computed iteratively assuming the leader to be conical with non uniform radius. The time domain EFIE code is used to compute the evolution of current in the lightning channel. The relation between leader charge and return stroke peak current is found to be linear. This is in satisfactory agreement with the most accepted formulation regarding the relationship between leader charge and return stroke peak current.

Keywords—return stroke; peak current; leader charge; self consistent model;

I. INTRODUCTION

Lightning return stroke modeling is extensively used to gain insight into various aspects of strike to ground and that to tall structures. They are also employed for evaluating the resulting electromagnetic fields and the efficacy of lightning protection systems.

Return stroke models fall into four broad categories – gas dynamic models, engineering models, transmission line models and antenna models [1]. The gas dynamic models or physical models provide the values of pressure, temperature etc when the current is specified. The engineering models provide the electric and magnetic fields provided the current distribution along the lightning channel is specified. These two categories of models are not designed to simulate the means of current evolution and hence not suitable to model the interaction between the lightning channel and transmission line [2]. The transmission line models or the distributed circuit models model the channel as a transmission line. Assumption of transmission line requires a suitable return conductor. In reality there is no return conductor, and if, for the sake of argument, the ground is considered as return conductor, the distance between the channel and ground (return) is very large. Moreover, TEM mode of propagation is assumed. Even though

TEM mode exists after a few tens of microseconds after the onset of the return stroke, in the initial stages this assumption is not valid.

In the case of antenna models or electrodynamic models, even though TEM mode is not assumed, the current is assumed [2]. Assuming current is equivalent to modeling the return stroke by a current source. Use of voltage sources as the excitation have also been attempted by researchers [3]. But they have been found to be equivalent to the current source approach [4]. The use of current sources demand a knowledge of the impedances involved. Various values for impedances have been suggested, but there is no general consensus among researchers regarding the value of impedance to be used for the current source. Recent measurements on UHV towers in Japan [5] show that peak currents and rise times are different for a strike to tower/ground and strike to phase conductor. To assess this scenario theoretically, a suitable return stroke model, that does not assume the value of current has to be employed. As explained earlier, the present categories of return stroke are not suitable for simulating the current evolution along the channel and for assessing the dependency of stroke current peak amplitude on the strike object.

A self consistent model for return stroke was proposed in [6] where the channel current and its distribution are not assumed. Based on the gradient along the leader channel, the distribution of the initial charge is deduced along with the radius of the corona sheath. Using a spark law for streamer section and first order arc equation for the leader, all the dynamic switching of the channel conductance is emulated. Time domain thin wire approach is employed to account for the dynamic field. With these, model simulates the spatio-temporal evolution of channel current quite naturally. This model has been validated for a strike to ground scenario, using the pattern of current evolution and signatures of far and near fields.

The above model was employed in another work to investigate on to the strike to simplified transmission line models [7]. The model could clearly depict the significant reduction in the peak amplitude of the stroke current when the stroke terminates on phase conductors.

Since charge is the dominant parameter that dictates the attachment of lightning to ground/structures, it is customary to relate the charge to the peak current of the return stroke, which is easily available in measurements [8]. Therefore, it would be worthwhile to scrutinize the above mentioned return stroke model with the leader charge and resulting peak stroke current for its versatility. Such an exercise would require reliable relation between the leader charge and resulting peak current. In view of the same, a very brief review of the leader charge models will be carried out below.

Golde was the first to suggest that the leader charge density decreases exponentially with increasing height above the ground. The relationship suggested by him [9] was

$$\rho_s = \rho_{s0} e^{-z/\lambda} \quad (1)$$

where λ is the decay height constant taken as 1000m and ρ_{s0} is the charge density at $z=0$ given by

$$\rho_{s0} = 4.36 \times 10^{-5} I_p \quad (2)$$

where I_p is the return stroke peak current in kA.

Eriksson [9], later on modified the relationship between ρ_{s0} and I_p considering the suggestion that 25 kA would correspond to a charge of 1C to

$$\rho_{s0} = 3.2 \times 10^{-6} I_p^{1.43} \quad (3)$$

Based on the observations of secondary peaks in some of Berger's waveforms and based on the argument that these secondary peaks are due to the reflections from the cloud end, Dellera and Garbagnati [10] came up with the relation

$$\rho_{s0} = 3.8 \times 10^{-5} I_p^{0.68} \quad (4)$$

Cooray, formulated a return stroke model [9] based on many simplifying assumptions regarding the corona sheath, duration of current in the corona sheath and regarding the temporal variation in current came up with the relation

$$\rho_{(\xi)} = a_0 \left(1 - \frac{\xi}{H - z_0}\right) G(z_0) I_p + \frac{I_p (a + b\xi)}{1 + c\xi + d\xi^2} H(z_0) \quad (5)$$

By employing a infinite disc model for the cloud charge distribution, leader with uniform radius and axial gradient, charge distribution along the channel for different cloud potentials were computed in [9]. Based on observations from Berger's experiments, Cooray came up with a linear relationship connecting the charge in the first 100 μ s to the peak current as shown in (6) which has been used in his return stroke model [9].

$$Q_{100\mu s} = 0.061 I_p \quad (6)$$

In Figure 1, all these models have been compared. Out of all these models, Cooray's formulation, which is based on established physical aspects of the phenomena, is currently the most accepted one.

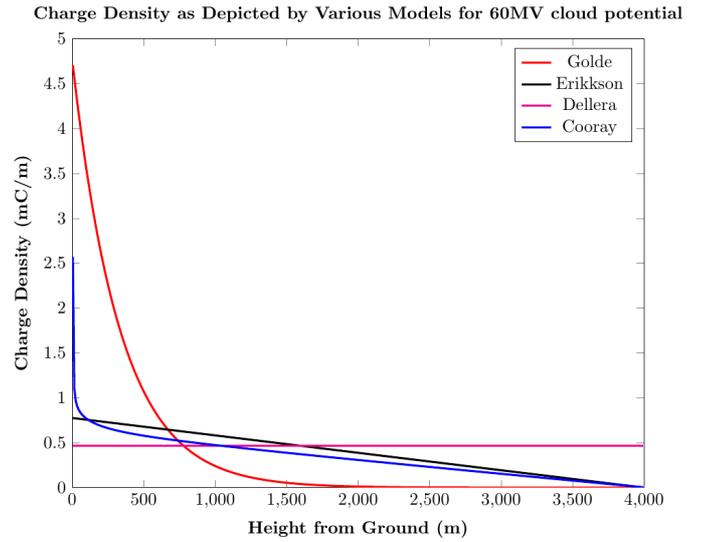


Figure 1: Charge Density for Various Models

The scope of the present work, as indicated earlier, is to scrutinize the peak current and leader charge relation as predicted by the self-consistent return stroke model proposed in [6].

In the following section, the method employed for evaluating the charge distribution along the leader, some essential details of return stroke model employed in the work, will be discussed first.

II. LEADER CHARGE MODEL

In this work, cloud charge distribution is modeled as ellipsoidal structures. Using the reported values of charge density and gradient in the cloud, the dimension of the ellipsoids are determined for every specified cloud base potential.

The leader channel is considered to be a thin structure both electrically and geometrically. The Charge Simulation Method (CSM) is used to discretize the cloud and the leader and the contributions of each element is computed. The charge distribution and radius of segments is computed iteratively assuming the leader segments to be conical with non uniform radius. The leader gradient was chosen as 6 kV/m and the streamer gradient was chosen as 500 kV/m based on the values suggested in [8]. The radial gradient was set to 15 kV/cm. Similar to the studies of Cooray [9], a 4 km long vertical channel is considered.

Figure 2 shows the charge distribution along the leader channel. This is also compared with the charge distribution obtained from Cooray's formulation.

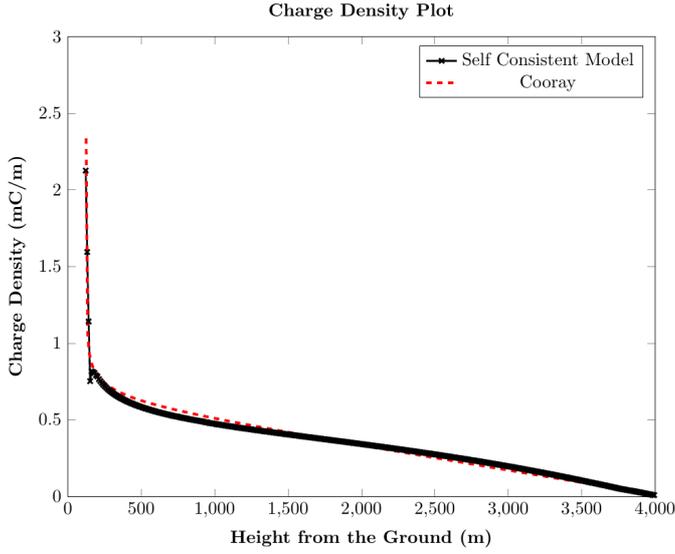


Figure 2: Charge Density Plot

It can be seen that the computed charge from the present approach match well with that given by Cooray's formulation. The radius profile obtained from the simulation of the current leader model is shown in Figure 3.

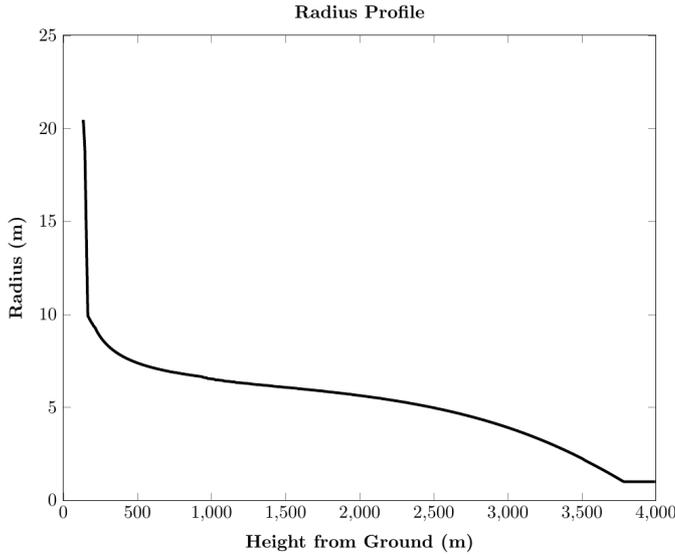


Figure 3: Radius Profile

Once the leader charge distribution is known, the streamer charges and radii are computed iteratively by considering the ambient field as the sum of field due to the cloud and the field due to the leader charges. Then the evolution of current is computed using the formulation in the self consistent model.

III. A SELF CONSISTENT RETURN STROKE MODEL

For the sake of completeness, the self consistent return stroke model proposed in [6], which is being employed in this work will be briefed below.

Since the leader channel is considered as a thin structure the electric field integral equation (EFIE) can be used to evaluate the current evolution in the leader.

$$\frac{\mu_0}{4\pi} \int_{C(\vec{r})} \begin{bmatrix} \frac{\hat{s}'}{R} \frac{\partial}{\partial t} I(s', t') \\ c \frac{\vec{R}}{R^2} \frac{\partial}{\partial s'} I(s', t') \\ -c \frac{\vec{R}}{R^3} q(s', t') \end{bmatrix} ds' + I(s', t') Z_s = \hat{s} \cdot \vec{E}(\vec{r}, t) \quad (7)$$

The right hand side of the equation represents the field due to charge on the leader, the contribution of the cloud and the image charges induced in the ground. The left hand side is the reaction field and the resistive drop along the channel. Point sectional collocation form of Method of Moments (MoM) is used so that the EFIE equation (7) can be solved as an initial value problem. The charges deposited on the leader channel and the contribution of the cloud forms the initial excitation.

The radius of the leader corona sheath, which varies with height along the channel, as explained earlier, is obtained by an iterative evaluation. During the evolution of the return stroke, the positive streamers invades into the original corona sheath and due to the process of neutralization, its velocity gets scaled from that in virgin air. As a result, the positive corona sheath evolves over time and hence all the corresponding matrix entries are calculated at every time step.

The dynamic channel conductance is modeled in two parts comprising of the arc regime and the spark regime. The spark regime, which represents the streamer section is modeled using Toepler's spark law represented by

$$R(t) = \frac{\alpha}{q_{(0)} + \int_0^t i dt} \quad (8)$$

where α and $q_{(0)}$ are constants which are scaled according to the spatial variation of spark resistance. The values of constants are chosen such that the resistance of the streamer is small at the bridging section where the streamer and leader meet and as the streamer progresses towards the ground, the resistance is increased. The conductance of the arc regime, which represents the leader section is modeled using the first order arc equation, which for rising currents is given by

$$\frac{dg(t)}{dt} = \frac{g_\infty(i) - g(t)}{\theta_r} \quad (9)$$

and for falling currents is given by

$$\frac{dg(t)}{dt} = \frac{-g(t)}{\theta_f} \quad (10)$$

where $g(t)$ is the arc conductance per unit length and time, θ_r and θ_f are the rising and falling time constants taken as $50 \mu\text{s}$ and $500 \mu\text{s}$ respectively. The switching from the streamer to leader happens when the energy level crosses 20 J/m .

IV. SIMULATION AND RESULTS

For a specified cloud potential, the leader charge distribution and the radius profile of the leader was computed as explained in Section II. With the radius of the leader segments as the input values, the current is computed as explained in Section III. The evolution of the current is shown in Figure 4 for various heights of the lightning channel.

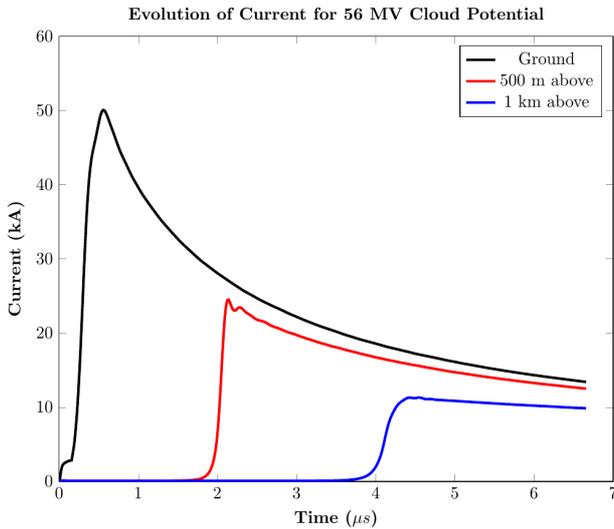


Figure 4: Evolution of Current

Simulations were carried out for different cloud potentials and the result is depicted in Figure 5.

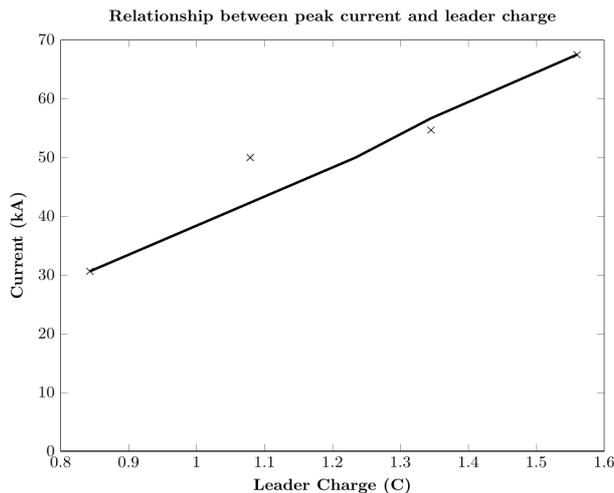


Figure 5: Peak Current and Leader Charge

The peak current is seen to be linearly correlated with the total leader charge in the leader channel with a correlation factor of 0.966 and the relationship can be expressed as

$$Q_{leader} = 0.0247 I_p \quad (11)$$

Under similar conditions, Cooray's model was used to check the validity of the relationship. The total leader charge as depicted by the Cooray's formulation in [9] gave a linear relationship with the peak current as in (12).

$$Q_{leader} = 0.0297 I_p \quad (12)$$

It can be seen that the relation between charge and current predicted by the self-consistent return stroke model agrees quite well with that given by Cooray et. al work. The small difference seen is attributed to slight over estimation of the channel radius employed in the present work. This issue has to be addressed in a future work.

V. CONCLUSION

The relation between the total leader charge and the peak of the return stroke was deduced from an self-consistent return stroke model (reported in [6]) and compared with the commonly employed model in the literature. It was shown that the agreement is quite good thereby providing a further validation to the self-consistent return stroke model of [6]. Based on the same, it is proposed that the above model can be reliably employed for investigation on the direct strike to objects.

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