



A simple physical model for self-triggered upward leaders from high-rise buildings

Ming Kit Chan, Mingli Chen* and Yaping Du
Department of Building Services Engineering
The Hong Kong Polytechnic University Hong Kong, China
[*mingli.chen@polyu.edu.hk](mailto:mingli.chen@polyu.edu.hk)

Abstract— This paper presents a simple physical model for an upward positive leader initiated from a tall grounded object. Characteristics such as the leader channel structure, leader initiation criteria, leader propagation speed, leader current and channel longitudinal electric field are described and modelled step-by-step. Other parameters including the evolution of leader line charge density, leader core conductance and radius, and leader corona sheath radius are also computed and evaluated. Particularly, the minimum corona sheath radius and corona charge density for leader initiation and stable propagation based on existing experiment data are proposed. The model may serve as a virtual lab to study the initiation of an upward leader at the top of a tall ground structure for tall building lightning protection analysis.

Keywords- lightning, upward leader, propagation model;

I. INTRODUCTION

Nowadays, many high-rise buildings, such as the Dubai Tower, have been built in metropolises around the world. When a thundercloud is growing above a tall building, more and more charges accumulate in the cloud and the ambient electric field strength below the charged cloud increases. If the electric field at the top of the building reaches the critical breakdown value, air breakdown will take place. Then, an upward leader is initiated and propagates towards the charged cloud. As the upward discharge can damage high objects, such as wind turbines [1,2], more researches have been rapidly focused on studying the initiation of upward lightning for protecting humans and animals [3-5].

Lightning physics modelling is important because most detection systems still cannot predict the initial lightning strike from a certain thundercloud. Modeling of the leader evolution of a lightning flash can help us to evaluate the physical thresholds for the phenomenon of lightning strikes to tall grounded objects. Recently, Oscar Diaz *et al.* [6] have made a general review of the existing engineering and physical models for positive leader channels and compared the models' simulation results with the experimental results of the long air gap electrical discharge. However, they have assumed the leader propagation speed is nearly constant.

This paper presents a simple physical model for studying the continuous propagation of a self-initiated upward positive leader. This approach is different from other existing models. We will also propose a method to estimate the minimum corona radius and charge density for leader initiation.

II. UPWARD LEADER MODEL

A thorough description of the self-triggered positive upward leader model, which includes both physical and engineering model features, is presented in this section. In this paper, we use the sign convection that a negative charge overhead produces a negative electric field on the ground.

A. Model Details

Although the thunderstorm cloud charge has already been recognized as a typical triple structure, in the present model, the charged cloud is simplified as a conductive plane with a potential, φ_{cloud} (e.g. -60 MV), at a height, H (e.g. 3000 m), resulting in a uniform cloud electric field between the cloud and the ground, E_{cloud} , (e.g. -20 kV/m). The electric field due to the corona space charge near the ground, E_{corona} , (e.g. -3 kV/m), can be determined based on the observed space electric field profile versus height under a thundercloud, which will be discussed later. For an upward leader triggered by other discharges such as a downward negative leader, it will be studied in a separate paper.

The upward positive leader (UPL) is simply assumed to propagate along a straight line without bending and it gets energy from an ambient electric field environment E_A , which is the uniform cloud electric field plus the corona electric field, i.e. $E_A = E_{cloud} + E_{corona}$.

The height dependent ambient field can be expressed as the following empirical equation [7]

$$E_A(z) = E_{cloud} + (E_{ground} - E_{cloud})e^{-z/L_C} \quad (1)$$

where E_{cloud} is the cloud charge produced electric field, E_{ground} is the electric field measured on ground, and L_C is the characteristic decay length of the space corona charge.

In our model, a tri-layer structure of the leader channel shown in Fig. 1 is introduced. While inside the black line is the leader core (R_L), the blue region is called the transition region (R_T). The third layer is the corona sheath zone (R_C). The leader core is full of quasi-neutral plasmas, positive and negative particles can move freely in the core and the lateral electric field, E_L is nearly zero. However, inside the surrounding corona sheath zone, the electric field is equal to the breakdown electric field, E_C . Outside the corona zone, the electric field reduces to the ambient electric field, E_A . To insert a thin non-quasi-neutral transitional layer between the core and the corona zone, the

lateral electric field can increase from nearly zero to the breakdown electric field continuously.

According to Chen *et al.* [8], the critical breakdown space electric field is proportional to the air density. This means it is a function of height above sea level:

$$E_c(z) = E_0 e^{-z/H_0}, \quad (2)$$

where E_0 equals 500 kV/m for the positive polarity is the critical breakdown value at ground level, z is the longitudinal direction above ground, and H_0 is a constant of 8400 m.

In addition to the leader structure model and the critical breakdown field, the leader charge distribution is calculated by the charge simulation method (CSM) [8] and the method of mirror charges. The building structure is simply assumed to be a regular structure which can be equivalent to a grounded thin metal rod.

For a long channel, the breakdown electric field is directed radially from it, the corona radius of the leader channel can be given by [9]

$$R_c(z) = \frac{\lambda_L(z)}{2\pi\epsilon_0 E_c(z)}, \quad (3)$$

where $\lambda_L(z)$ is line charge density distribution along the leader channel.

To simulate the leader propagation speed, we assume the driving force of the moving leader tip is due to the average of the corona sheath electric field, E_C , and the electric field inside the channel core, E_L , that may either increase or decrease when the leader grows steadily.

Besides this, inside the streamer-leader transition region, the air heating is assumed to be isobaric ($\Delta P = 0$) and the total mass of the tip is conserved ($\Delta M = 0$). Based on conservation of energy, the gain in kinetic energy of charged particles inside the leader tip is equal to the difference of the injected electrical energy and the energy losses due to heat, vibration and friction. Therefore, the speed of the leader tip can be express as

$$v_{tip} = \sqrt{\int \left[\frac{F(\eta(E_c + E_L) - 2E_{Dr})}{M} \right] dl}, \quad (4)$$

where η represents the heat and vibrational energy lost, F is the Faraday constant equals 96485.3365 C/mol, $M = \sum_i (w_i M_i)$ is the effective molar mass among ions composition, and $E_{Dr} \approx 5.58 \times 10^{-18} (n_e \ln A) / T_e$, is the Dreicer field [10], $\ln A$ is the Coulomb logarithm, and T_e is the electron temperature. Note: $Q/M = eN/M = eN_A/M = F/M$, where Q is the total injected charges.

Then, the leader propagation speed equals

$$v_L = \frac{v_{tip}}{1 + \tau_d / \tau_a}, \quad (5)$$

where τ_a and τ_d are the three-body attachment time scale, and delay time for a new leader segment to cross the stream-to-leader transition region, respectively. τ_d is also defined as the time needed to heat the channel up to 5000 K.

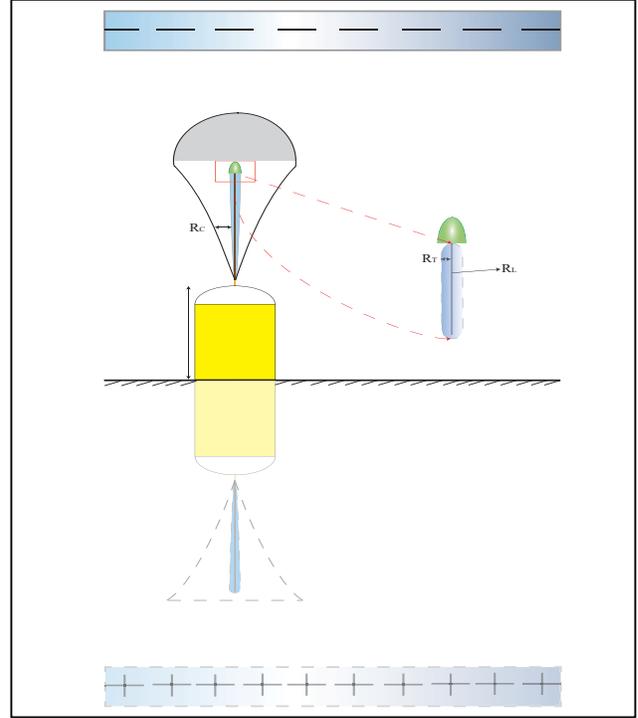


Figure 1. The channel structure of an upward positive leader (not in scale)

From [11, 12], the gas dynamic model simulation result showed that $\tau_d \propto \rho_a^{-2}$, where ρ_a is the ambient air density. In our model, we assume

$$\tau_d = \tau_{d0} e^{2z/H_0}. \quad (6)$$

Furthermore, we have applied the semi-empirical equation proposed by M. Bazelyan and P. Raizer [13] to estimate the average leader electric-field:

$$E_L I_L = E_L \lambda_L v_L = b, \quad (7)$$

B. Minimum initial channel length of a steady upward leader

When the leader is just triggered ($t = 0$), $E_c \gg E_L$ and E_{Dr} , and the leader speed is given by

$$v_{L0} \approx \sqrt{\left(\frac{l_0 F \eta E_c}{M} \right) / (1 + \tau_d / \tau_a)}$$

Experimental results have showed the minimum starting leader speed is nearly 10 kms⁻¹ [15,16]. By using the parameter value shown in Table I, we can estimate the minimum initial leader length at ground level as

$$l_0 = 0.10463 \text{ m} \approx R_{C0}.$$

Here, we have assumed the minimum corona radius (R_{C0}) is equal to the minimum initial leader length at ground level.

TABLE I. PARAMETERS FOR SIMULATION OF AN POSITIVE UPWARD LEADER.

Symbol	Table column subhead	Value	Unit
M	Effective molar mass	73	g/mol
H	Simulation height	3000	m
H_b	Length of the building height	150.5/303	m
E_{cloud}	Electric field due to the cloud charge	-20	kV/m
E_{C0}	Breakdown electric field at ground level	500	kV/m
T_e	Electron temperature	20000	K
τ_{a0}	Three-body attachment time scale at ground level	0.1	μs
τ_{d0}	Delay time at ground level	1	μs
η	Heat and vibrational energy lost	0.175	Not applicable

As a result, the minimum initial leader line charge density at ground level is equal to

$$\lambda_{L0} = 2\pi\epsilon_0 E_{C0} R_{C0} = 2.91024 \mu C/m.$$

The minimum height (H_b) of a building to trigger an upward leader under E_A is then estimated based on

$$(\int^{H_b} E_A(z) dz) / E_C \geq R_{C0}.$$

III. SIMULATION RESULTS AND DISCUSSION

Based on the models presented in the previous sections, simulations were done for a self-triggered upward positive leader on a building with and without a space charge layer.

- Background conditions:

$$E_{cloud} = -20 \text{ kV/m}, E_{ground} = -3 \text{ kV/m}, H = 3 \text{ km},$$

$$L_C = 250 \text{ m}.$$

- Step length: $\Delta l = 0.3 \text{ m}$, $\Delta t_i = \Delta l / v_{Li}$

A. Constant ambient field

In case A, the ambient field remains constant and the minimum self-triggered height is estimated as $H_b = 150.5 \text{ m}$. The evolutions of leader channel line charge density, corona radius, propagation speed, current, electric field, leader conductance and core radius with time and height are presented in Figs. 2 - 8 respectively. As shown in Fig. 2, the charge along the channel is increasing. This is because the leader electric field is decreasing with time and the potential difference between the leader channel and the environment keeps increasing. Simulation results also show the leader propagation speed reaches its maximum value at a certain tip height, and then starts to decrease.

B. Space charge layer included

In case B, the ambient field is no longer constant but changing with height. The minimum self-triggered height is estimated as $H_b = 303 \text{ m}$ which is higher than case A. The evolutions of leader channel line charge density, corona radius are presented in Figs. 9 - 10 respectively. Although the shape of the figures is very similar to case A, the amount of charge along the channel is relatively smaller.

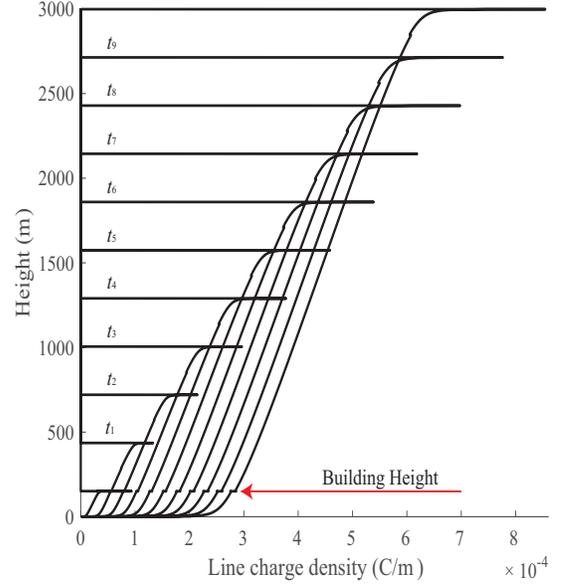


Figure 2. Line charge density evolution of the leader for Case A

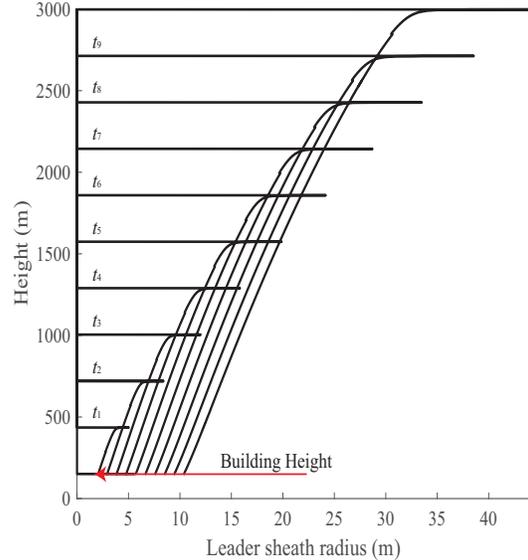


Figure 3. Corona sheath radius evolution of the leader for Case A

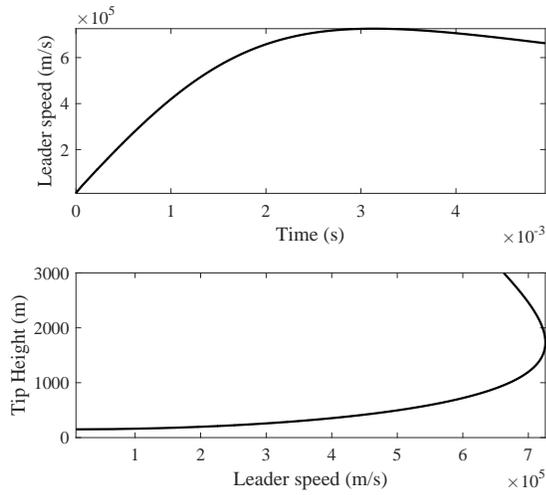


Figure 4. Leader speed evolution for Case A

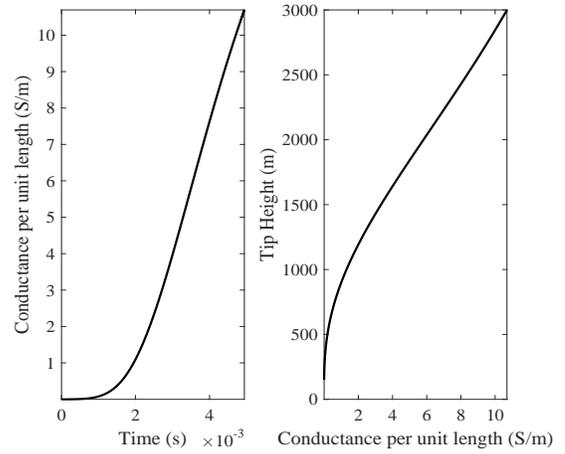


Figure 7. Leader conductance per unit evolution for Case A

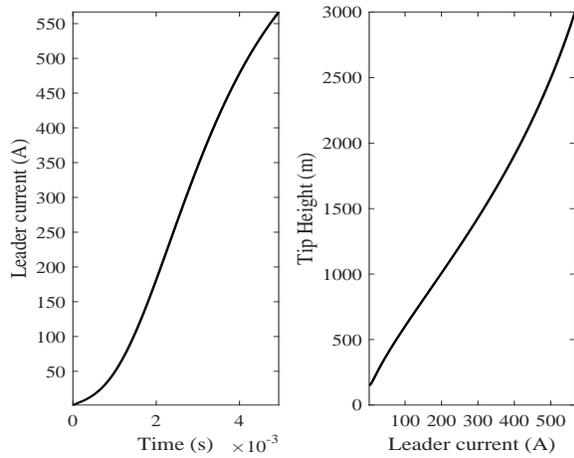


Figure 5. Leader current evolution for Case A

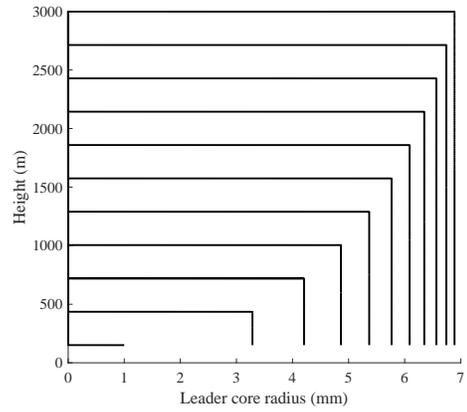


Figure 8. Leader core radius evolution for Case A

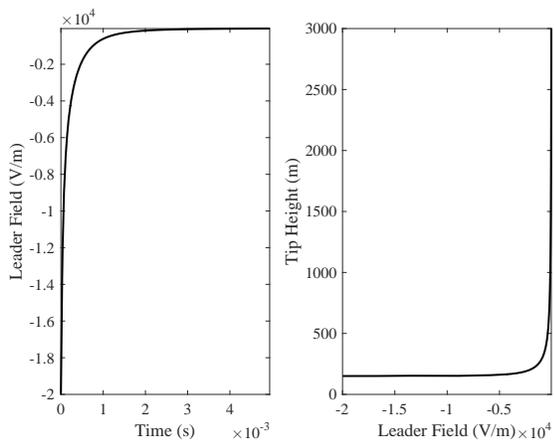


Figure 6. Leader electric field evolution for Case A

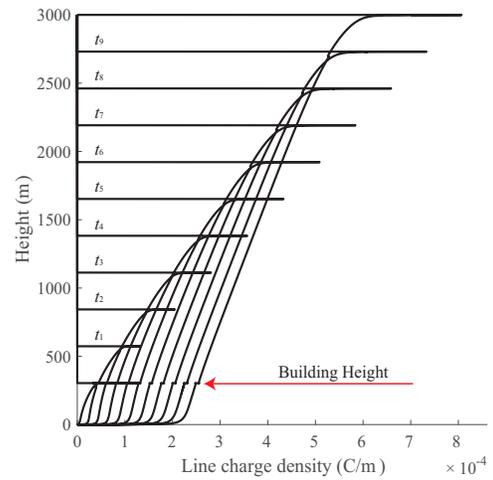


Figure 9. Line charge density evolution of the leader for Case B

