Study of Earth–Ionosphere Waveguide Effect on Lightning Pulse with Ray Theory

Zilong QIN and *Mingli CHEN

Department of Building Service Engineering
The Hong Kong Polytechnic University
Hong Kong, China
*mingli.chen@polyu.edu.hk

Baoyou ZHU

School of Earth and Space Sciences
University of Science and Technology of China
Hefei 230026, China
zhuby@ustc.edu.cn

Abstract— **In this paper we present an half analytic electromagnetic wave model for the study of a lightning-produced VLF/LF pulse propagating in the earth-ionosphere waveguide (EIWG). The property of the lightning pulse propagating in the ionospheric D region is retrieved in frequency domain by using the Transfer Matrix Method coupled with presumed electron density and electron-collision rate profiles. With the retrieved reflection coefficients, a Ray Theory based transfer function from the lightning source to the receiver is derived. The propagation attenuation through spherical ground with presumed conductance is also calculated. The model is applied to the data from a lightning detection network in central China. The simulation results match perfectly with the ground waves and sferics observed by the network, indicating that the model is an efficient tool to investigate the interaction of the lightning pulse with the ionospheric D region and the earth's magnetic field.**

Keywords—lightning pulse; ionospheric D region; ray theory

I. INTRODUCTION

Lightning is the most intensive electrical discharge process in the terrestrial space. It emits a wide band of electromagnetic waves from several Hz to a few GHz, which the most of radiation energy is concentrated in VLF and LF band. The ionospheric D region (the bottom of earth's ionosphere) has the lowest electron density and highest electron-neutral collision rates, forming a transition plasma region that only interferes with VLF/LF waves. The high-pass effect of the ionospheric D region makes the lightning VLF/LF wave an efficient tool to investigate the electron density distribution there. In this study we propose a ray-theory-based model for calculation of the propagation effect of lightning pulses within EIWG. By tuning the modelling result against experimental data, the space-time characteristics of corresponding ionospheric D region can be retrieved.

The first work on retrieving the electron densities of lower ionosphere by lightning sferics was a mode-theory-based model developed by Cummer and Inan [1], which achieved the path averaged electron density from the source to receiver by matching the spectrum of modeled and observed sferics. Sooner this model was improved by Cheng and Cummer[2, 3] to investigate the lightning induced ionospheric perturbations.

Meanwhile, Hu and Cummer [4] established an improved 2D cylinder FDTD model to simulate the lightning-generated EM field, and verified its accuracy with previous mode-theory-based model. Through this model, Han and Cummer [5-7] studied the variation characteristics of ionosphere D region during day and night time at mid-latitude. A full wave EM model of ionosphere reflection with special concerning on Fresnel diffraction was developed by Jacobson and Shao [8-10]. The reduction effects on electron density of lower ionosphere from thunder storms had been revealed by Shao, Jacobson and Lay [11, 12] through this model coupling with observations from Los Alamos Sferic Array (LASA)[13].

In this study we propose a ray-theory-based model to simulate the lightning pulses in EWIG. The ray theory (wave hop theory) is based on geometric optics which traces every path effects of a number of several discrete ray paths. The ionospheric D region is treated as magnetized, anisotropic, collisional, and cold horizontal stratified plasma. The transfer matrix method in [14] is applied to calculate the propagation properties of lightning sferics in this dispersive D region. The model is then applied to lightning sferics recorded by a lightning detection network in central China [15, 16] and favorable results are obtained.

II. METHODOLOGY

The spherical expanding lightning source in EIWG can be treated as a summation of infinite plane waves in different
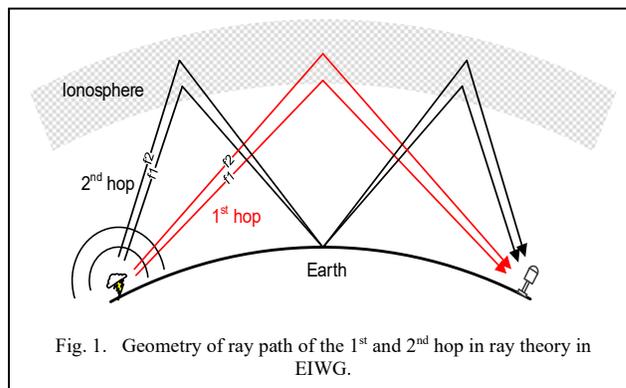

Fig. 1. Geometry of ray path of the 1st and 2nd hop in ray theory in EIWG.

directions. Each time-domain plane wave can be decomposed into the summation of different frequencies. We first derive the transfer function for each frequency for each direction plane wave, and then superimpose them at receiver point to get the total fields there. However, only one frequency among all plane waves in one direction will arrive at the receiver since the ionosphere is dispersive.

A. The parameters of ionospheric D region

In this model, the lower ionosphere is set as a transition cold plasma region with constant earth's magnetic field. The whole region is horizontally stratified and is split into several homogeneous slabs. Each slab has different electron density and collision frequency. Only the effect of electrons is considered.

The electron density profile and electron-neutral collision rate profile are adopted from Volland [17]:

$$N_e = 3 \times 10^8 \times e^{\beta \times (z - z_0)} \ m^{-3} \qquad (1)$$

$$v_{en} = 5 \times 10^6 \times e^{-0.15 \times (z - 70)} \ s^{-1} \qquad (2)$$

Where β represents the sharpness and z_0 refer to the overall altitude of the distribution of N_e.

Due to the Lorentz force by magnetic field and damping effect by collisions, this region is anisotropic and attentional to VLF and LF waves. All these effects can be perceived in a susceptibility tensor matrix [18].

B. The reflection coefficients of lower ionosphere

To calculate the dispersive and reflection properties of ionospheric D region, the transfer matrix method proposed by Nagano [14] is adopted. The transfer matrix method is widely used in EM propagation problem in dispersive media [19]. First, the Maxwell equations can be converted into another form [20]:

$$\frac{d}{dz} e = -jk_0 T e \qquad (3)$$

$$e = \begin{bmatrix} E_x \\ -E_y \\ Z_0 H_x \\ Z_0 H_y \end{bmatrix} \qquad (4)$$

Where T is the state matrix for each slab. By applying the boundary conditions at each interface between 2 neighbor slabs to the transfer functions derived from the matrix T, the field vector at each slab can be retrieved by a group of multiplications from the top in downward direction while e is known at the top boundary. Due to that the only energy input into the ionosphere is from the incident wave, it is reasonable to assume that only 2 upward waves remain at a relatively high altitude, which are known as 'propagated wave' and 'evanescent waves' [20]. The fields at each layer can be retrieved recursively from the bottom boundary conditions:

$$A \prod_{k=2}^{m-1} K_k e_P + B \prod_{k=2}^{m-1} K_k e_E = e_{TM}^I + e_{TE}^I + e_{TM}^R + e_{TE}^R \qquad (5)$$

The reflection coefficients can then be determined:

$$\begin{bmatrix} {}_\parallel R_\parallel & {}_\perp R_\parallel \\ {}_\parallel R_\perp & {}_\parallel R_\perp \end{bmatrix} \begin{bmatrix} E_{TM}^I \\ E_{TE}^I \end{bmatrix} = \begin{bmatrix} E_{TM}^R \\ E_{TE}^R \end{bmatrix} \qquad (6)$$

C. The Transfer funtion derived from Ray Theory.

In Ray Theory, EM fields at receiver can be retrieved by superimposing a few numbers of discrete induced field rays through different paths, which may be reflected by ionosphere 1 or 2 times (generally named 1st hop and 2nd hop) [21-24]. The lightning discharge is treated as a vertical dipole on ground, which emits EM fields in a spherical expansion manner. The whole geometry is shown in Fig.1.

We use a transfer function derived from the effects of ray path combining with the strength factor recommended by International Telecommunications Union (ITU) [25], and only vertical E field is concerned. For a vertical antenna, E field at ground for the 1st hop is:

$$E_1(\omega) = \frac{2E_0(\omega)[sin(\theta)]^2 \ {}_\parallel R_\parallel F_i F_t F_r \left(e^{-jk_0[sin(\theta)d + cos(\theta)2h]} \right)}{L} \qquad (7)$$

Where d is the direct distance from source to receiver, h the height from ground to bottom boundary of the lower ionosphere, θ the incident angle, and L the total length of radio path. When the distance d, reflection coefficient ${}_\parallel R_\parallel$ and frequency of E_0 are given, θ and L can be found by a phase swapping process [8-9]. Factor F_i is the ionospheric focusing factor due to the curvature of ionosphere, which is also a function of d. F_t and F_r are the antenna factors, which start taking effect only when ground distance is further than 1500 km with a 60 km reflection height. Assuming them equal to 1 will always be valid at a shorter distance. Similarly, the transfer function of the field of the 2nd hop is:

$$E_2(\omega) = \frac{2E_0(\omega)[sin(\theta)]^2 \left({}_\parallel R_\parallel R_G \ {}_\parallel R_\parallel + {}_\parallel R_\perp R_G \ {}_\perp R_\parallel \right) F_i F_t F_r \left(e^{-jk_0[sin(\theta)d + cos(\theta)4h]} \right)}{L} \qquad (8)$$

However, there is no pure TM waves existed in the ionosphere. Reflected radio waves must have both TM and TE components. So $E_2(\omega)$ should include both TM and TE waves induced by lower ionosphere.
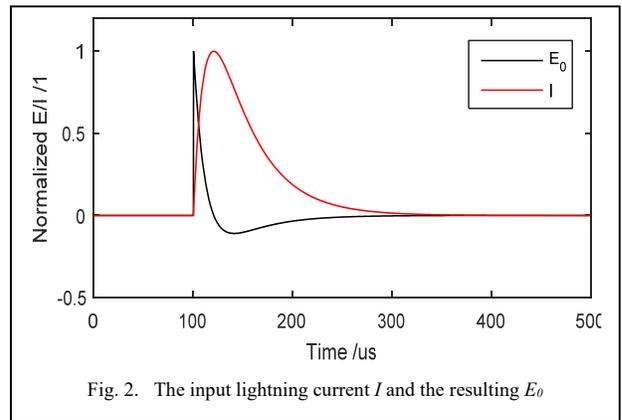

Fig. 2. The input lightning current I and the resulting E_0

The transfer function of ground wave is calculated by Fock's diffraction equation [26], which is the summation of a few numbers of discrete modes solved by mode equation.

$$E_G(\omega) = \frac{E_0(\omega)We^{-jk_0d}}{d} \qquad (9)$$

$$W = 2e^{-j\frac{\pi}{4}}\sqrt{\pi x}\sum_{S=1}^{\infty}\frac{e^{-jxt_S}}{t_S-q^2} \qquad (10)$$

The earth surface is assumed to be uniform ground with $\sigma = 0.005$ s and $\varepsilon = 15$. The current waveform of lightning discharge used in the model is from Heidler [27]:

$$I(t) = e^{-77140\times\frac{t}{3}} - e^{-248900\times\frac{t}{3}} \qquad (11)$$

$$E_0(t) = \frac{L}{4\pi\varepsilon_0 c^2}\frac{dI}{dt} \qquad (12)$$

First, the input field $E_0(t)$ is transformed into frequency domain. The source is spherically expanding in EIWG. Different frequency components transverse by different path with different transfer functions. Then, the total field at receiver in time domain is achieved with an inverse Fourier transform:

$$E(t) = \int_0^\infty \left(E_G(\omega) + \sum_{i=1}^{\infty}E_i(\omega)\right)e^{j\omega t}d\omega \qquad (13)$$

At distances shorter than 1000 km, only the 1st hop or 2nd hop is visible. So, in this modeling only the 1st hop is considered.
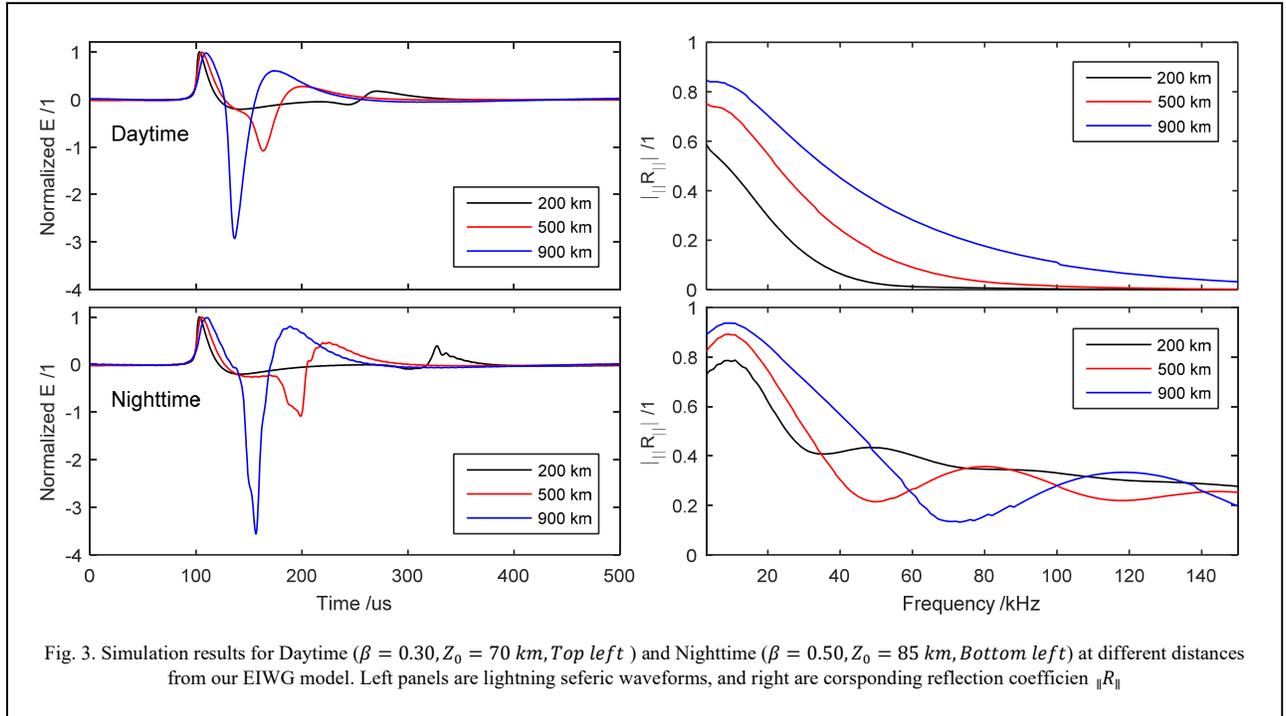
III. MODEL RESULTS

To validate the model, simulations and comparisons with observations are done for several cases. For all cases, the input earth magnetic field is $4.8\times10^{-5}T$ with a dip angle of $48°$, which is from the International Geomagnetic Reference Field (IGRF)[28]. The input lightning current I and the resulting incident wave E_0 is shown in Fig.2. The simulation is performed in the frequency band of 3 kHz ~ 150 kHz, since the ionosphere theory is inaccurate at higher frequency and the ray theory is invalidate at lower frequency.

A. General simulations for daytime and nighttime

With an electron density profile of β =0.30 and Z_0 =70 km for daytime, and of β =0.50 and Z_0 =85 km for nighttime, the properties of both the ionosphere reflection coefficient and the lightning pulse waveform at different distance on ground are simulated. The wave propagation direction is assumed from east to west and has a 90° angle to the earth's magnetic field.

The results are shown in Fig.3. As can be seen from the figure, in the daytime, the ionospheric D region shows a strong low-pass feature at all 3 distances. As the distance goes further, the ratio of the sferics to ground wave gets larger. At nighttime, the ionosphere shows a relative high reflection rate at high frequency and the waveform of the sferics is shaper than that at daytime.

The high reflection rate at nighttime is due to the lower collision rate at higher altitudes. As a result, there are less damping effects applying on electrons accelerated by EM field. Meanwhile, a sharper transition region at night also shortens

Fig. 3. Simulation results for Daytime ($\beta = 0.30, Z_0 = 70\ km, Top\ left$) and Nighttime ($\beta = 0.50, Z_0 = 85\ km, Bottom\ left$) at different distances from our EIWG model. Left panels are lightning seferic waveforms, and right are corsponding reflection coefficien $_\|R_\|$

the penetration depth of the waves, making the attenuation decrease.

B. Simulations for observed sferics

During the summer 2012, we built a local lightning detection networks in central China. Each station consists of a monopole vertical antenna with an 800Hz-450 kHz pre-amp circuit and an uninterrupted data logging system. Since then, a great number of lightning waveform have been recorded with by the network. Some of them are used for simulations and comparisons in this paper.

Shown in Fig.4 are simulation results for some observed lightning sferics. In the figure, the left 3 panels are for daytime cases and the right 3 for nighttime, from near to far distance. The angle of the wave propagation direction to the earth magnetic field depends on the location of the lightning discharge. For each case, the best match in waveform between the simulation and observation is found by tuning the profile of the ionosphere electron density in ranges of β = 0.1~0.8 and Z_0 =65~ 95 km. Only the 1st hop is considered, as the spatial inhomogeneous of the ionosphere D region may introduce inaccuracies into the 2nd hop.

As can be seen from the figure, the observed waveform matches perfectly with the simulated one for each case for both the daytime and nighttime. The nighttime cases show a more complex pattern comparing to the daytime cases. Apparently, more high frequency components exist in the sferics in nighttime.

IV. CONCLUSION

In summary, we have proposed a ray-theory-based model for simulation of the properties of lightning sferics propagation in EIWG in VLF/LF bands. Simulations and comparisons with observations show that this newly proposed model is more effective with higher accuracy than existing models in lighting sferic study.
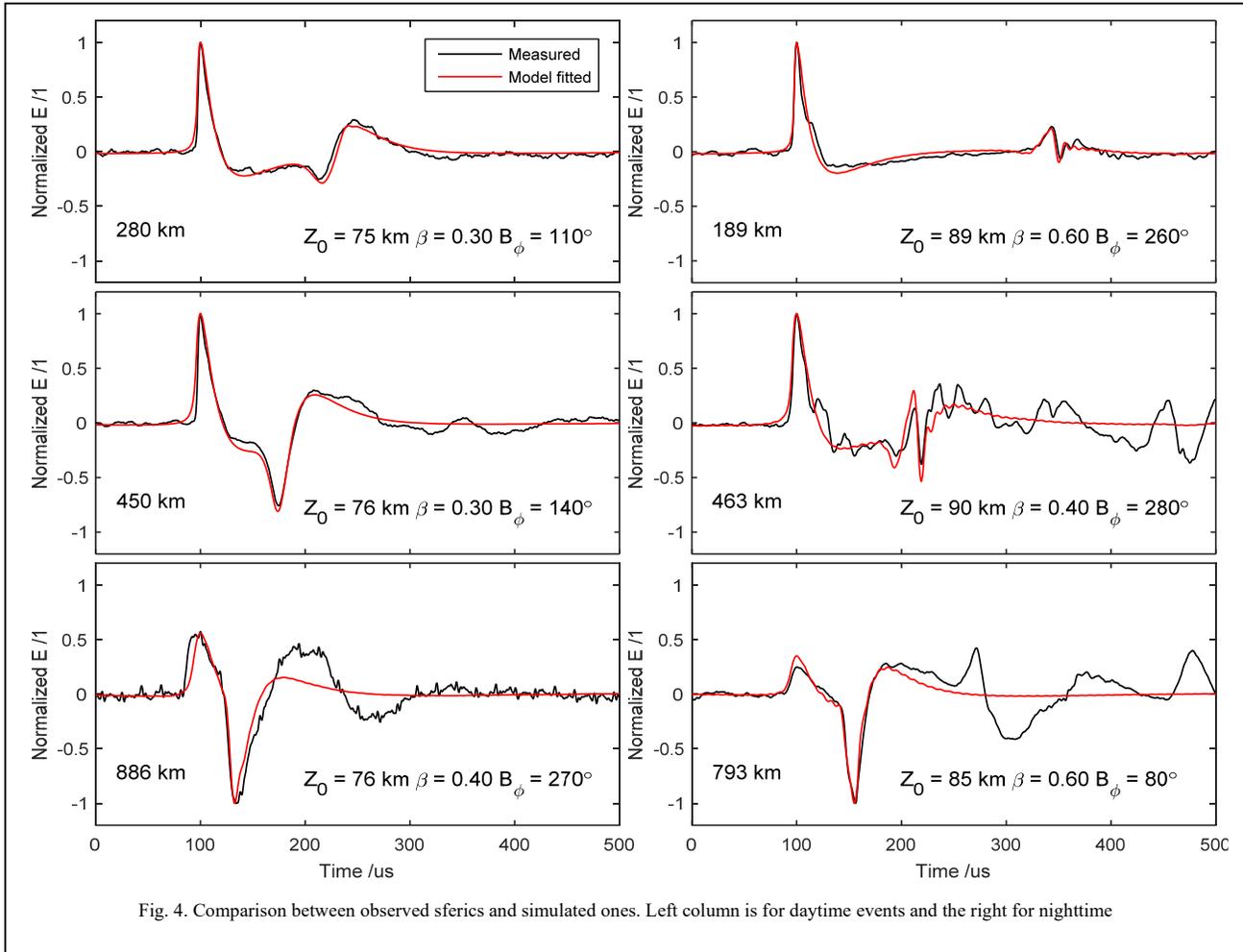
REFERENCES

Fig. 4. Comparison between observed sferics and simulated ones. Left column is for daytime events and the right for nighttime

[1] S. A. Cummer, "Lightning and ionospheric remote sensing using VLF/ELF radio atmospherics," stanford university, 1997.

[2] Z. Cheng, and S. A. Cummer, "Broadband VLF measurements of lightning-induced ionospheric perturbations," Geophysical Research Letters, vol. 32, no. 8, 2005.

[3] Z. Cheng, S. A. Cummer, D. N. Baker, and S. G. Kanekal, "Nighttime D region electron density profiles and variabilities inferred from broadband measurements using VLF radio emissions from lightning," Journal of Geophysical Research, vol. 111, no. A5, 2006.

[4] W. Hu, and S. A. Cummer, "An FDTD model for low and high altitude lightning-generated EM fields," Antennas and Propagation, IEEE Transactions on, vol. 54, no. 5, pp. 1513-1522, 2006.

[5] F. Han, and S. A. Cummer, "Midlatitude daytime D region ionosphere variations measured from radio atmospherics," Journal of Geophysical Research, vol. 115, no. A10, 2010.

[6] F. Han, and S. A. Cummer, "Midlatitude nighttime D region ionosphere variability on hourly to monthly time scales," Journal of Geophysical Research, vol. 115, no. A9, 2010.

[7] F. Han, S. A. Cummer, J. Li, and G. Lu, "Daytime ionosphericDregion sharpness derived from VLF radio atmospherics," Journal of Geophysical Research, vol. 116, no. A5, 2011.

[8] A. R. Jacobson, X.-M. Shao, and R. Holzworth, "Full-wave reflection of lightning long-wave radio pulses from the ionosphericDregion: Numerical model," Journal of Geophysical Research, vol. 114, no. A3, 2009.

[9] A. R. Jacobson, X.-M. Shao, and E. Lay, "Time domain waveform and azimuth variation of ionospherically reflected VLF/LF radio emissions from lightning," Radio Science, vol. 47, no. 4, 2012.

[10] X.-M. Shao, and A. R. Jacobson, "Model simulation of very low-frequency and low-frequency lightning signal propagation over intermediate ranges," Electromagnetic Compatibility, IEEE Transactions on, vol. 51, no. 3, pp. 519-525, 2009.

[11] E. H. Lay, and X.-M. Shao, "Multi-station probing of thunderstorm-generated D-layer fluctuations by using time-domain lightning waveforms," Geophysical Research Letters, vol. 38, no. 23, 2011.

[12] X.-M. Shao, E. H. Lay, and A. R. Jacobson, "Reduction of electron density in the night-time lower ionosphere in response to a thunderstorm," Nature Geoscience, 2012.

[13] X.-M. Shao, M. Stanley, A. Regan, J. Harlin, M. Pongratz, and M. Stock, "Total Lightning Observations with the New and Improved Los Alamos Sferic Array (LASA)," Journal of Atmospheric & Oceanic Technology, vol. 23, no. 10, 2006.

[14] I. Nagano, M. Mambo, and G. Hutatsuishi, "Numerical calculation of electromagnetic waves in an anisotropic multilayered medium," Radio Science, vol. 10, no. 6, pp. 611-617, 1975.

[15] Q. Zilong, "The Observation of Ionospheric D-layer based on the Multi-station Lightning Detection System (in Chinese)," University of Science and Technology of China, Hefei, 2014.

[16] Q. Zilong, Z. Baoyou, L. Fanchao, M. Ming, and M. Dong, "Using time domain waveforms of return strokes to retrieve the daytime fluctuation of ionospheric D layer(in Chinese)," Chinese Science Bulletin, no. 7, pp. 654-663, 2015.

[17] H. Volland, Handbook of atmospheric electrodynamics: CRC Press, 1995.

[18] K. G. Budden, The propagation of radio waves: The theory of radio waves of low power in the ionosphere and magnetosphere: Cambridge University Press, 1988.

[19] W. C. Chew, Waves and fields in inhomogeneous media: IEEE press New York, 1995.

[20] K. G. Budden, "The Numerical Solution of the Differential Equations Governing the Reflexion of Long Radio Waves from the Ionosphere. II," Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, vol. 248, no. 939, pp. 45-72, 1955.

[21] S. Pal, "Numerical modelling of VLF radio wave propagation through earth-ionosphere waveguide and its application to sudden ionospheric disturbances," arXiv preprint arXiv:1503.05789, 2015.

[22] K. Davies, Ionospheric radio: IET, 1990.

[23] T. G. Wolf, "Remote Sensing of Ionospheric Effects Associated with Lightning Using Very Low Frequency Radio Signals," 1990.

[24] W. L. Poulsen, "Modeling of very low frequency wave propagation and scattering within the Earth-ionosphere waveguide in the presence of lower ionospheric disturbances," to the Department of Electrical Engineering.Stanford University, 1991.

[25] I. Recommendation, "684-3 „Prediction of field strength at frequencies below 150 kHz," ITU, Geneva, 2002.

[26] V. A. Fock, Electromagnetic diffraction and propagation problems: Pergamon Press, 1965.

[27] F. Heidler, "Traveling current source model for LEMP calculation." pp. 157-162.

[28] NOAA, "International Geomagnetic Reference Field."