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Surge Behavior on the Single Conductor with a Discontinuity

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Abstract— **This paper addressed surge propagation on a vertical line with a discontinuity. A formula of surge reflection coefficient expressed by specially-defined surge impedance was introduced, which is similar to that of a conventional transmission line. The formula for the discontinuity with a lumped resistance was also presented. With these formulas, surges on the line with the discontinuity can be evaluated directly. The numerical results for the discontinuity with and without a lumped resistance were presented in this paper. These results were compared with those obtained from a circuit analysis approach – PEEC method. It is found that these results matched well. The average errors are generally less than 1%.**

Keywords-surge; conductor; reflection coefficient; lightning

I. Introduction

The analysis of lightning surges on a vertical structure is of great significance in lightning protection. In the past several decades, many numerical methods have been developed to analyze electromagnetic phenomena during a lightning strike, such as the finite-element method (FEM), the partial-element-equivalent-circuit (PEEC), the finite-difference time-domain method (FDTD) and so on. However, until today analysis of lightning surges on a vertical conductor with a discontinuity is not well addressed.

It is known that a surge current attentuates during its propagation along a vertical conductor even if the condcuotr is perfectly conductive[1-5]. The authors in [2] proposed a "scatter theory" to explain the current attenuation. The attenuation is caused by the reflected surge along the vertical line. This theory is difficult to apply for the surge propagation across a line discontinuity. In [6] reflection coefficient, which is similar to that used for a transitional transmission line, was introduced for the discontinuity of a vertical conductor. This coefficient is determined by three distinct surge impedances of the line [7-8]. They are (1) incident wave impedance $z_{in}(t)$, (2) transmitted wave impedance $z_{tr}(t)$ and (3) reflected wave impedance $z_{ref}(t)$. These impedances are defined by their corresponding voltages and currents, as follows:

$$z_{in}(t)=\frac{v_{in}(t)}{i_{in}(t)} \tag{1a}$$

$$z_{tr}(t)=\frac{v_{tr}(t)}{i_{tr}(t)} \tag{1b}$$

$$z_{ref}(t)=\frac{v_{ref}(t)}{i_{ref}(t)} \tag{1c}$$

However, there is no simple method available to calculate these surge impedances as well as their corresponding voltages and currents [9-13]. It would be difficult to analyze the surge propagation at the discontinuity.

This paper propose a novel method for analyzing surge propagation on the vertical conductor with a discontinuity. Reflection coefficient of a surge at the discontinuity is determined using single-definition surge impedances. Such impedances can be calculated easily and efficiently. Surge voltages and currents after surge reflection and refraction at the discontinuity can be then analyzed. Moreover, surge progatation on the line with a lumped resisitive load at the discontinuity is also presented. The proposed method is validated by comparing the simulation results obtained with the PEEC method.

II. Reflection Coefficient of a Surge at a Discontinuity

Shown in Fig. 1 is a configuration of a vertical conductor with a discontinuity at z_0 under a direct lightning stroke. A current surge $I_1(z,t)$ propagates towards the discontinuity, and generates a reflected surge $I_2(z,t)$ and a transmitted surge at the discontinuity. The transmitted surge is decomposed into two components: $I_1^{'}(z,t)$ and $I_2^{'}(z,t)$. These two components are selected in such a way that

$$\begin{aligned} I_1(z_0,t)&=I_1'(z_0,t)\\ I_2(z_0,t)&=I_2'(z_0,t)\end{aligned} \tag{2}$$

at the position of the discontinuity z_0 .Therefore, the current on two sides of the discontinuity is expressed

$$I(z,t)=\begin{cases} I_1(z,t)+I_2(z,t) & z\le z_0\\ I_1'(z,t)+I_2'(z,t) & z\ge z_0\end{cases} \tag{3}$$

With the current in (3) potential (voltage) on the conductor in a free space is then expressed by

$$\phi(z,t)=\int_z^{+\infty} j\omega\mu_0\int_0^{+\infty} G(r,r')I(r',t)p(r',t)dr'dr \tag{4}$$

where $p(r',t)$ is the coefficient of current attenuation at position r' and time t. In (4) both Green function $G(r,\vec{r})$ and retarded current $I(r',t)$ are given by

$$\begin{aligned} G(r,\vec{r}) &= \frac{1}{4\pi}\cdot\frac{e^{-jk|r-r'|}}{|r-r'|} \\ I(r',t) &= I(0,t-\frac{r'}{c}) \end{aligned} \tag{5}$$

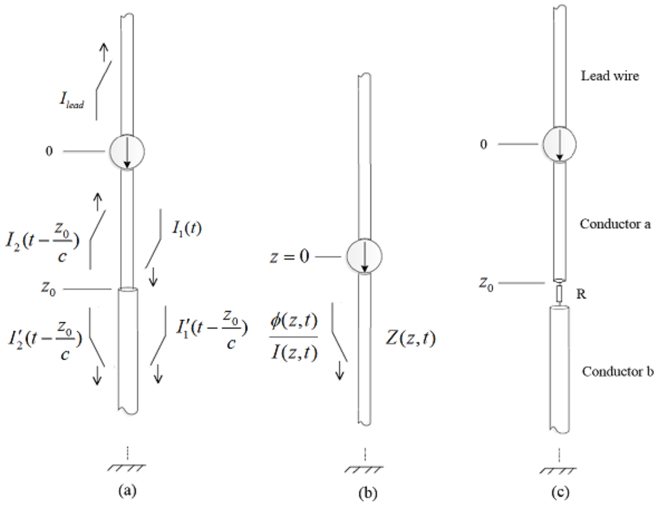

Fig. 1 Configurations of a vertical line with a discontinuity
(a) Current waves resulted from a discontinuity.
(b) Definition of surge impedance.
(c) Configuration for the line with a lumped resistor

Substituting (2) in (4) yields a voltage on the conductor, which is expressed with four current components. At the position of the discontinuity z_0, these four components are grouped into two voltages $\phi_1(z_0,t)$ and $\phi_2(z_0,t)$, as follows:

$$\phi(z_0,t) = \phi_1(z_0,t) + \phi_2(z_0,t) \tag{6}$$

and

$$\begin{aligned} \phi_1(z_0,t) = & \int_{x_0}^{+\infty} j\omega\mu \int_0^{x_0} G(l,l')I_1(l',t)p_1(l',t)dl'dl + \\ & \int_{x_0}^{+\infty} j\omega\mu \int_{x_0}^{+\infty} G(l,l')I_1'(l',t)p_1'(l',t)dl'dl \end{aligned} \tag{7a}$$

$$\begin{aligned} \phi_2(z_0,t) = & \int_{x_0}^{+\infty} j\omega\mu \int_0^{x_0} G(l,l')I_2(l',t)p_2(l',t)dl'dl + \\ & \int_{x_0}^{+\infty} j\omega\mu \int_{x_0}^{+\infty} G(l,l')I_2'(l',t)p_2'(l',t)dl'dl \end{aligned} \tag{7b}$$

Clearly, both $\phi_1(z,t)$ and $\phi_2(z,t)$ are two voltage surges on the upper conductor, which are associated with current pairs of $I_1(z,t)$ and $I_1'(z,t)$, $I_2(z,t)$ and $I_2'(z,t)$, respectively. Note that radius of the lower conductor has little influence on the current, voltage and surge impedance on the upper conductor. Both $\phi_1(z,t)$ and $\phi_2(z,t)$ at $z=z_0$ can be expressed approximately by

$$\begin{aligned} \phi_1(z_0,t) &= \phi_a(z_0,t) \\ &= I_1(z_0,t)\cdot Z_a(z_0,t) \end{aligned} \tag{8a}$$

$$\begin{aligned} \phi_2(z_0,t') &= -\phi_a(0,t-z_0/c) \\ &= -I_2(z_0,t)\cdot Z_a(z_0,t') \end{aligned} \tag{8b}$$

Similarly, on the lower conductor we have

$$\begin{aligned} \phi_1'(z_0,t) &= \phi_b(z_0,t) \\ &= I_1'(z_0,t)\cdot Z_b(z_0,t) \end{aligned} \tag{9a}$$

$$\begin{aligned} \phi_2'(z_0,t) &= \phi_b(z_0,t-z_0/c) \\ &= I_1'(z_0,t)\cdot Z_b(z_0,t-z_0/c) \end{aligned} \tag{9b}$$

where $Z_a(z_0,t)$ and $Z_b(z_0,t)$ are respectively surge impedances of conductors a and b at $z=z_0$ without any discontinuity, as shown in Fig. 1(b). These impedances can be calculated with an iterative method given in [8].

At $z=z_0$ the following equation holds:

$$\phi_1(z_0,t)+\phi_2(z_0,t)=\phi_1'(z_0,t)+\phi_2'(z_0,t) \tag{10}$$

Substituting (8) and (9) into (10) yield an equation for $I_1(z_0,t)$ and $I_2(z_0,t)$. The reflection coefficient of current α is then derived, as follows:

$$\begin{aligned} \alpha_I(t) &= \frac{I_2(z_0,t)}{I_1(z_0,t)} \\ &= \frac{Z_a(z_0,t)-Z_b(z_0,t)}{Z_a(0,t-\frac{z_0}{c})+Z_b(0,t-\frac{z_0}{c})} \end{aligned} \tag{11}$$

If there is a load connected between upper and lower conductors, such as a resistor, the equation for voltage balance at $z=z_0$ is revised, as follows:

$$\phi_1(x_0,t)+\phi_2(x_0,t)=\phi_1'(x_0,t)+\phi_2'(x_0,t)+\Delta\phi(x_0,t) \tag{12}$$

In (12) $\Delta\phi$ is introduced to balance the voltage at both sides of discontinuity. In case of resistance R is connected at the discontinuity, $\Delta\phi$ is expressed by

$$\Delta\phi = [I_1(z_0,t)+I_2(z_0,t)]\times R \tag{13}$$

The reflection coefficient of the surge current is modified to be

$$\alpha_I(t) = \frac{I_2(z_0,t)}{I_1(z_0,t)} = \frac{Z_a(z_0,t)-Z_b(z_0,t)-R}{Z_a(0,t-\frac{z_0}{c})+Z_b(0,t-\frac{z_0}{c})+R} \tag{14}$$

III. SURGES PROPAGATING BEYOND THE DISCONTINUITY

The evaluation procedure for surges on both sides of the discontinuity is similar. We analyze the surges on the upper conductor only in this section, that is, $z \le z_0$. In this case, there are two waves propagating on upper conductor. Both total voltage and current on the upper conductor are given by

$$\begin{aligned} I(z,t) &= I_1(z,t)+I_2(z,t) \\ \phi(z,t) &= \phi_1(z,t)+\phi_2(z,t) \end{aligned} \quad \text{for } z \le z_0 \tag{15}$$

Similarly, both voltage and current on the lower conductor, that is, $z > z_0$, are given by:

$$I(z,t) = I_1'(z,t) + I_2'(z,t)$$
$$\phi(z,t) = \phi_1'(z,t) + \phi_2'(z,t) \quad \text{for } z > z_0 . \qquad (16)$$

A. Surge voltage $\phi_1(z,t)$ and current $I_1(z,t)$

Surge current $I_1(z,t)$ propagates downwards on the upper conductor *a* (a single conductor) as if there is no discontinuity at $z \le z_0$. Thus for position $z \le z_0$ and time *t*, both current and voltage on the upper conductor are given by:

$$I_1(z,t) = I_1(0, t - \frac{z}{c}) \times p_a(z,t) \qquad (17a)$$

$$\phi_1(z,t) = I_1(0, t - \frac{z}{c}) \times p_a(z,t) \times Z_a(z,t) \qquad (17b)$$

Both $p_a(z,t)$ and $Z_a(z,t)$ are calculated with the iterative method presented in [8].

B. Surge voltage $\phi_2(z,t)$ and current $I_2(z,t)$

Surge current $I_2(z,t)$ propagates upwards on the upper conductor as if there is a current source placed at $z = z_0$. The source current at $z = z_0$ is given by

$$\begin{aligned} I_2(z_0,t) &= \alpha_I(t) I_1(z_0,t) \\ &= I_1(z_0, t - \frac{z_0}{c} - \frac{d}{c}) \times p_a(d, t - \frac{z_0}{c}) \end{aligned} \qquad (18)$$

Thus, for position $z \le z_0$ and time *t*, both current and voltage are expressed by

$$\begin{aligned} I_2(z,t) &= I_2(z_0,t) \times p_a(d, t - \frac{z_0}{c}) \\ \phi_2(z,t) &= I_2(z,t) \times Z_a(d, t - \frac{z_0}{c}) \end{aligned} \qquad (19)$$

where distance $d = |z - z_0|$.

IV. Simulation Results and Comparisons

Computer simulations have been performed for the surges on a conductor system illustrated in Fig. 1. The systems consists of three lines segments. The upper one is the lead wire for the current source, which represents a lightning channel. The current source represents a lightning return stroke. It is of a ramp waveform. A discontinuity is located between the middle and lower segments. The upper and middle segments are 60m long, and have a radius of 5mm. The lower segment is 120m long and has a radius of 10mm. Surge reflection and transmission will be observed at the discontinuity. At the discontinuity, a lumped resistor is connected, which has a resistance of 0 or 500 ohm.

The MATLAB codes using the proposed procedure (VTL method) have been developed to calculate surges on the segments above and below the discontinuity. The results are compared with those obtained using the PEEC method. In each case, surge voltages and currents at two positions, that is, 45m, 75m from the current-source point.

Fig. 2 and Fig. 3 show both currents and voltages on the line calculated with the proposed method (VTL method) and PEEC method under a ramp-wave current source. In this case the discontinuity the lumped resistance is set to be zero. Fig. 4 shows the reflection coefficient of current the discontinuity. It is found that the results from two methods match well.

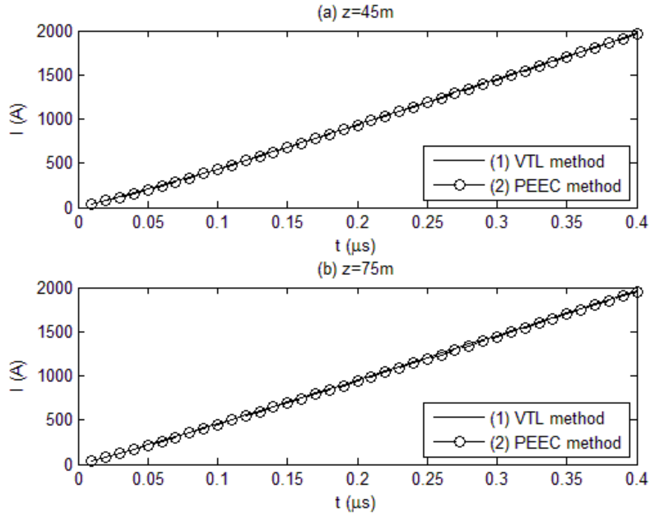

Fig 2 Currents calculated by the VTL method and PEEC method with R=0ohm
(a) at position on the left side of discontinuity (z=45m)
(b) at position on the right side of discontinuity (z=75m)

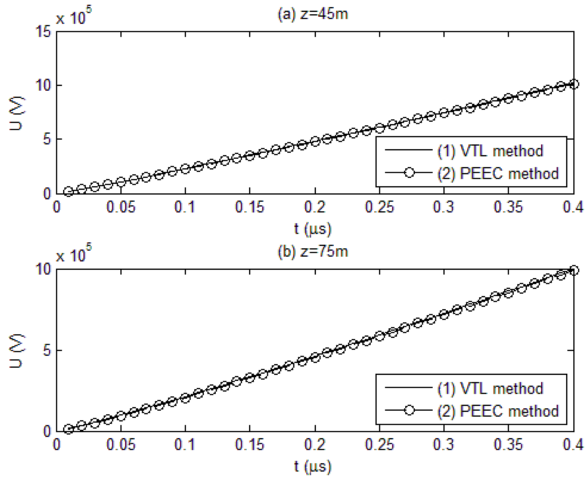

Fig 3 Voltages calculated by the VTL method and PEEC method with R=0ohm
(a) at position on the left side of discontinuity (z=45m)
(b) at position on the right side of discontinuity (z=75m)

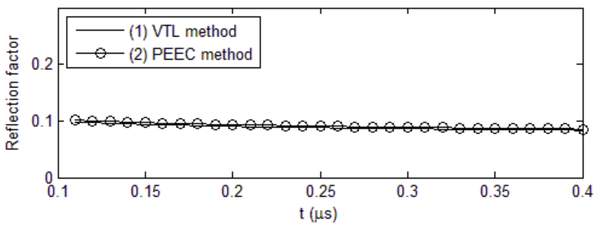

Fig. 4 Reflection coefficient at the discontinuity calculated by the proposed method and PEEC method

Fig. 5 and Fig. 6 show both currents and voltages on the line calculated with the proposed method (VTL method) and PEEC method under a ramp-wave current source. In this case the lumped resistance at the discontinuity is set to be 500ohm. Fig. 7 shows the reflection coefficient of current at the discontinuity.

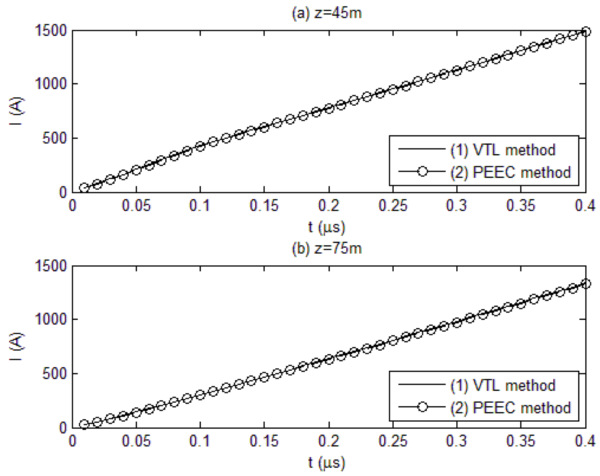

Fig 5 Currents calculated by the VTL method and PEEC method with R=500ohm
(a) at position on the left side of discontinuity (z=45m)
(b) at position on the right side of discontinuity (z=75m)

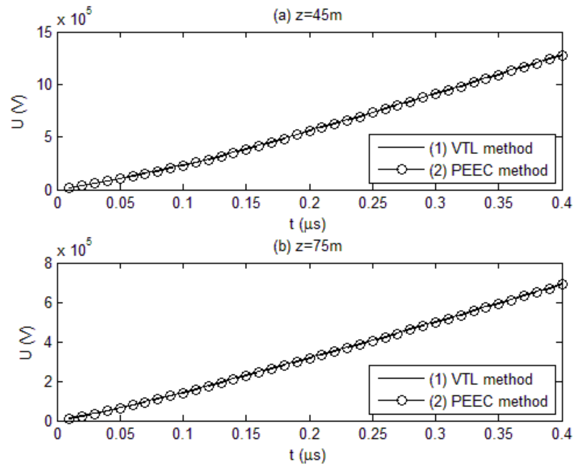

Fig 6 Voltages calculated by the VTL method and PEEC method with R=500ohm
(a) at position on the left side of discontinuity (z=45m)
(b) at position on the right side of discontinuity (z=75m)

Table 1 shows the average errors of the voltages, currents and impedances calculated with two different methods. It is found that these errors are generally less than 1%. It is found the average errors of currents and voltages are slightly larger than the error of surge impedances, which is below 0.4% generally. A possible reason is that calculation result from PEEC method may contain some noises. At the early time period these noises may not be negligible, as both currents and voltages are really small. As time goes on, the contribution from these noise is very small, and can be negligible.

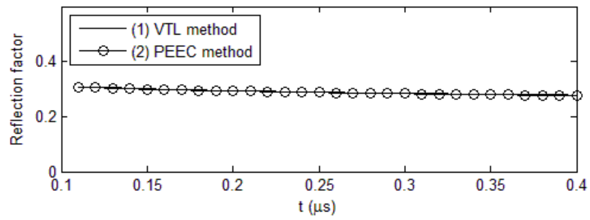

Fig. 7 Reflection coefficient at the discontinuity calculated by the proposed method and PEEC method

Table.1 Average errors in each configuration

Resistance	Current	Voltage	Surge impedance
R=0	1.0%	0.9%	0.1%
R=500 Ohm	0.6%	1.1%	0.4%

CONCLUSION

This paper addressed surge propagation behavior at the discontinuity for a thin-wire conductor. A time-dependent surge reflection coefficient was proposed based on a special-defined surge impedance. Compared with traditional transmission line theory, this reflection coefficient is influenced by both the position and time of a source. The formula for the discontinuity with a lumped resistance was also presented. With these formulas, surges on the line with the discontinuity can be evaluated directly. The numerical results for the discontinuity with and without a lumped resistance were presented in this paper. These results were compared with those obtained from a circuit analysis approach – PEEC method. It is found that these results matched well. The average errors are generally less than 1%.

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